Advanced Control Systems

Detection, Estimation, and Filtering

Graduate Course on the Mechanical Engineering PhD Program

Spring 2012/2013

2^{nd} Problem Set

This is the second problem set, of a series of five, in the Detection, Estimation, and Filtering Course. This problem set is composed by five problems related to basic concepts on statistics, probability, linear systems, and stochastic processes, introduced in the first weeks of the course.

Problem 1.

Assume that two trains **X** and **Y** arrive at a train station randomly in the time interval between 8h and 8h 30 minutes. The train **X** stops 4 minutes and the train **Y** stops 5 minutes, to let passengers to get off the train and to admit new ones. Assume that both trains arrive from different locations and no common trajectories were shared before the arrival to the aforementioned station.

a) What are the probability density functions of the trains **X** and **Y** arrival to the train station? Provide a graphical interpretation for your result.

b) What is the joint probability density function of the arrival to the station?

c) What is the probability of train **X** to arrive between 8 h and 8 h 15 minutes and train **Y** between 8 h 15 minutes and 8 h 30 minutes?

d) Compute the probability of train **X** to arrive before train **Y**?

e) Compute the probability on the simultaneous presence of both trains in the station (in different platforms)?

d) In the case that both trains meet at the train station, what is the probability of train **X** to arrive before train **Y**? Provide an interpretation to the result obtained.

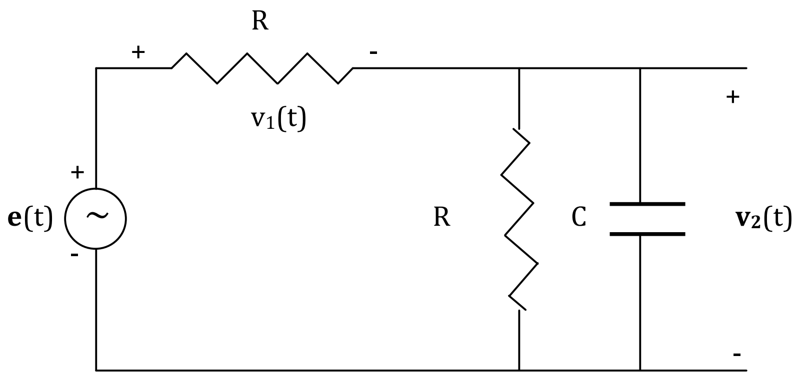
Problem 2.

In order to detect an underwater vehicle, a SONAR sensor is installed on a pier. Assume that this sensor provides measurements of the range to the target **r** and the bearing **ψ**, relative to a given reference direction, both disturbed by some physical, electrical and electro-magnetic phenomena. Considering the planar (2D) version of this problem, the measurements are given by

$$\begin{cases} \mathbf{r} = \sqrt{\mathbf{x}^2 + \mathbf{y}^2} \\ \psi = \arctan(\mathbf{y}/\mathbf{x}) \end{cases},$$

where **x** e **y** are the Cartesian coordinates of the target. Discuss how the joint probability density functions, in Cartesian coordinates $p_{xy}(\mathbf{x},\mathbf{y})$ and in sensor (polar) coordinates $p_{r\psi}(\mathbf{r},\psi)$, are related.

Problem 3.



In the circuit depicted in the previous figure, **e**(t) is a telegraphic signal that can assume the values 1 V e -1 V, being both equi-provable. The duration of each pulse is described by a Poisson process.

a) Show that the auto-correlation of this ergodic process is given by

$$\psi_{ee}(\tau) = \exp(-2|\tau|\lambda),$$

where l is the rate of signal commutations.

b) Assuming that $1/RC=\alpha$, what are the transfer functions from the input **e**(t) to the outputs $\mathbf{v_1}(t)$ and $\mathbf{v_2}(t)$?

c) What are the power spectral densities of the signals $\mathbf{v_1}(t)$ e $\mathbf{v_2}(t)$, respectively? Compute the variance of those signals.

Problem 4.

Assume that a system can be described by the first order differential equation

$$\frac{d\mathbf{x}(\mathbf{t})}{dt} + 2\mathbf{x}(\mathbf{t}) = \mathbf{u}(\mathbf{t}),$$

where u(t) is the external input.

a) What is the order of this system? Express this model in the form of an equivalent state space model?

b) Is the system linear? Justify your answer resorting to the properties of linear systems.

c) Compute the power spectral density of $\mathbf{x}(t)$, when the input corresponds to a signal with autocorrelation $\boldsymbol{\psi}_{uu}=5\boldsymbol{\delta}(t)$, where $\boldsymbol{\delta}(t)$ is the Dirac impulse.

d) What is the power spectral density of $\mathbf{x}(t)$when the input is a zero mean Gaussian white noise with power spectral density $\sigma^2=2$.

e) Validate the results obtained in this problem resorting to a set of simplified simulations in MATLAB/SIMULINK.

Problem 5.

Consider the following two discrete-time first-order systems, the first stable and the other unstable, described by the difference equations:

Stable $x(t+1) = 0.9x(t) + \xi(t)$, and Unstable $x(t+1) = -3.5x(t) + \xi(t)$.

For both systems, the scalar random initial state is described by a Gaussian $x(0) \approx N(3,2)$, and the scalar random disturbance $\xi(t)$ is described by a Gaussian white sequence $\xi(t) \approx N(0,1)$. Furthermore, $x(0)$ and $\xi(t)$ are independent for all $t = 0,1,2,\ldots$.

a) For each system evaluate analytically the evolution of the mean and variance.

b) Plot versus time the mean trajectory plus and minus one standard deviation.

c) Comment on the uncertainty propagation in stable and unstable systems.

Problem set issued on April 4th 2013

Solutions due in April 18th 2013

Bom trabalho ;)

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IST, April 4th 2013