Advanced Control Systems

Detection, Estimation, and Filtering

Graduate Course on the Mechanical Engineering PhD Program

Spring 2012/2013

3rd Problem Set

This problem set consists on the detailed synthesis and analysis of a optimal estimation problem. Physical insight for the solutions obtained is important.

Problem

Consider the problem of estimating the state variables of a Direct-Current (DC) motor described by the transfer function

$$\mathbf{G}(\mathbf{s}) = \frac{\boldsymbol{\varpi}(\mathbf{s})}{\mathbf{u}(\mathbf{s})}\frac{\boldsymbol{\theta}(\mathbf{s})}{\boldsymbol{\varpi}(\mathbf{s})} = \frac{\mathbf{a}}{\mathbf{s}(\mathbf{s}+\mathbf{b})},$$

where $\boldsymbol{\theta}$(t) is the motor rotor angle, $\boldsymbol{\omega}$(t) is the angular velocity, and **u**(t) is the command voltage at the motor input. For numerical computations consider a=100 rad s^{-1} V^{-1} and b=30 s^{-1}.

Part 1

1) Write a state model for the motor described by the transfer function **G**(**s**).

2) Resorting to the step-invariant discretization method obtain a discrete time equivalent model, given the sampling period h=0.01s.

3) Characterize the resulting model in terms of stability and invariance. How are these properties for the continuous and discrete time versions related? And in the general case of non-linear systems?

4) Simulate the discrete time systems for an interval of 10 seconds, considering that the input is given by **u**(k)=sin(2 π k h) V, k=1…1000. The motor starts the experiment at rest with the rotor angle at 30 π /180 rad. This test, given the absence of stochastic phenomena is usually denominated as "sanity check".

Parte 2

After extensive laboratory tests it was found that the angular position is corrupted by zero mean white Gaussian noise with variance $(0.1\ \pi/180)^2$ rad^2. The velocity is corrupted by zero mean white Gaussian noise with variance $(0.05\ \pi/180)^2$ $(rad\ s^{-1})^2$.

a) Compute how this stochastic process evolves, given that the motor starts at rest with the initial angular position described as a random Gaussian variable with expected value $30\ \pi/180$ rad and variance $(0.5\ \pi/180)^2$ rad^2.

b) Show the evolution of the state variables resorting to 10 MATLAB simulations. Comment on the results obtained.

Part 3

Given the uncertainties previously described, a stationary Kalman filter should be designed to obtain optimal estimates on the state variables. To achieve that purpose consider that a angular sensor is installed on the motor axis, providing measurements with null mean error and variance
$(5\ \pi/180)^2$ rad^2.

a) Design a block diagram, as detailed as possible, to describe the Kalman filter and its connection to the "real" system.

b) Compute the evolution of the expected values of the state variables considered.

c) Compute the evolution of the state estimate error covariance, for the state variables considered.

d) Implement in MATLAB the proposed block diagram and simulate for a reasonable time interval the evolution of the motor and of the estimator. Consider also the original continuous time model of the motor. Comment on the results.

e) Compute the stationary Kalman filter gains and compare them with the results previously obtained. Discuss the results obtained.

Parte 4

Assume that a more accurate sensor becomes available, now with a variance $(0.1\ \pi/180)^2$ rad^2.

f) Based on the expressions obtained in c) and e) compute the new values for the estimation error covariance and the Kalman filter gains. Discuss the results.

g) Obtain the transfer functions from the angular measurement inputs to the angular position estimates. Discuss the stability and bandwidth of the systems obtained, resorting to a Bode diagram, for the two cases studied previously.

Solutions due in May 13th 2013

Bom trabalho ;)

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IST, May 1st 2013