DYNAMIC STOCHASTIC FILTERING, PREDICTION, AND SMOOTHING

Instituto Superior Técnico

PhD Program in Electrical Engineering and Computers

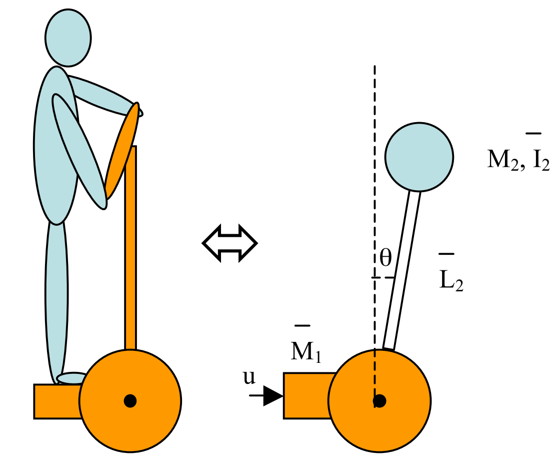
Area of Systems, Decision, and Control

Spring Semester 2009/2010

3rd and 4th Problem Sets

These problem sets exposes the students to a joint state and parameter nonlinear estimation problem to be solved resorting to Multiple Model Adaptive Estimation techniques. The discrete time problem to be tackled consists on the design of an estimator for a linearized system working with a nonlinear controller, in a closed-loop configuration.

Problem



Suppose that you are recruited to help on the development of a human transportation system (HTS), as represented above (see in www.segway.com a commercially solution available). The vehicle is composed by motors, sensors, batteries, and micro-controllers, able of transporting a human being and can be modeled as an inverted pendulum installed on a mobile platform.

The platform has a nominal weight of 20 Kg and the user is described by a concentrated mass **M_2**, at a height of 1 m, with an inertia moment of 100 $Kg.m^2$.

The system dynamics for the angle relative to the vertical ($\boldsymbol{\theta}$) can be described by

$$\left(m_{12} - \frac{m_{11}m_{22}}{m_{12}}\right)\ddot{\theta} + m_{11}g\tan(\theta) - M_2\bar{L}_2\sin(\theta)\dot{\theta}^2 = u,$$

where

$$m_{11} = \bar{M}_1 + M_2,$$
$$m_{12} = M_2\bar{L}_2\cos(\theta),$$
$$m_{22} = M_2\bar{L}_2^2 + \bar{I}_2,$$

where **g**=10 m/s^2 is the acceleration of the gravity and **u** is the actuation force in the system (generated by an electrical motor). The HTS research and development team implemented already a simulated version of the nonlinear regulator in discrete-time to control the angle relative to the vertical. The sensor package available includes an inclinometer sensor $\boldsymbol{\theta}$ and a rate-gyro sensor $\dot{\boldsymbol{\theta}}$, both sampled at the common sampling period of 0.01 s. The regulator was implemented in two phases:

i) resorting to the *feedback linearization* technique the following controller was proposed:

$$u = \left(\bar{m}_{12} - \frac{\bar{m}_{11}\bar{m}_{22}}{\bar{m}_{12}}\right)u^* + \bar{m}_{11}g\tan(\theta_m) - \bar{M}_2\bar{L}_2\sin(\theta_m)\dot{\theta}_m{}^2,$$

where

$$\bar{m}_{11} = \bar{M}_1 + \bar{M}_2,$$
$$\bar{m}_{12} = \bar{M}_2\bar{L}_2\cos(\theta_m),$$
$$\bar{m}_{22} = \bar{M}_2\bar{L}_2^2 + \bar{I}_2.$$

A nominal weight of 75 Kg was considered in the controller design and the controller inputs are the measurements obtained by the inclinometer ($\boldsymbol{\theta}_m$) and the rate-gyro ($\dot{\boldsymbol{\theta}}_{\mathbf{m}}$). Access to reliable measurements of the aforementioned quantities is assumed, i.e. the simplifying assumption $\boldsymbol{\theta}_m \approx \boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}_{\mathbf{m}} \approx \dot{\boldsymbol{\theta}}$ was considered. The resulting closed loop system corresponds to a double integrator

$$\ddot{\boldsymbol{\theta}} = \mathbf{u}^*.$$

ii) resorting to the *linear state feedback* technique, the following control law was proposed:

$$\mathbf{u}^* = -\mathbf{k}_1\boldsymbol{\theta}_{\mathbf{m}} - \mathbf{k}_2\dot{\boldsymbol{\theta}}_{\mathbf{m}},$$

where the gains were selected to be k_2=3 and k_1=9/4.

Important remark:

The system must be used by human beings with weights between 50 Kg and 100Kg!

Are the previous assumptions reasonable? How to improve the system's performance on the range of weights required?

The first opinion that it is required to be provided is on the choice of the sensors. In the next table some data on the sensors available and respective prices are summarized:

Inclinometer			*Rate-gyro*	
Noise intensity (σ^2)	Price		Noise intensity (σ^2)	Price
30	1		10	3
3	10		1	30
.3	100		.1	300

a) Can you propose a systematic methodology for the choice of the sensor package to be used, such that its price is minimized with guaranteed stability of the control system?

b) Resorting to the Simulink model provided, obtain the time evolution of the state variables for the values of masses in the interval of interest. Comment on the results.

c) Considering the equilibrium point $\boldsymbol{\theta} = 0$, $\dot{\boldsymbol{\theta}} = 0$, and u=0 find a discrete-time linear system that describes the dynamics of the HTS, for small disturbed angles.

Suggestion: maintain the explicit dependence on the parameter $\mathbf{M_2}$.

d) Design a set of 11 Kalman filters corresponding to the hypotheses

$$\mathbf{M}_2 = \{50,55,60,65,70,75,80,85,90,95,100\} \text{ [Kg]}.$$

e) Design an open loop Multiple Model Adaptive Estimator to obtain estimates $\hat{\boldsymbol{\theta}}$ and $\hat{\dot{\boldsymbol{\theta}}}$ of the variables $\boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}$, respectively, to be used by humans in the aforementioned interval of weights. Suggest realistic initial conditions for the estimator.

f) Test the substitution of the state variables $\boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}$ in the controller by $\hat{\boldsymbol{\theta}}$ and $\hat{\dot{\boldsymbol{\theta}}}$, obtained with the MMAE, when an human being with 95 Kg uses the HTS. Comment on the results.

g) Test the modification on the controller of the nominal value of $\mathbf{M_2}$ by the empiric value

$$\mathbf{M}_2^{\mathbf{MMAE}} = \sum_{\mathbf{i=1}}^{11} \mathbf{P_i M}_{2,i},$$

where $\mathbf{P_i}$ is the probability associated with model i and $\mathbf{M_{2,i}}$ is the hypothetical value of $\mathbf{M_2}$ in model i, respectively. Consider the case of a 60 Kg user. Comment the results obtained for the different sets of sensors considered.

Note: A SIMULINK file is in annex that corresponds to a nonlinear simulation of the proposed system, incorporating already the nonlinear controller described above. Furthermore, a script for the initialization of some relevant parameters is also provided.

Paulo Oliveira

IST, April 28th 2010

Due date: May 27th 2010

Bom trabalho ;)