REF. NO. KF #12

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STOCHASTIC ESTIMATION

Numerical Example:
Sensor Tradeoffs

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MOTIVATION

To examine what happens when one has to choose one out of three sensors upon

(a) Kalman filter
(b) Estimation accuracy

- Special example of sensor tradeoff problem

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DESIGN ISSUES

- Should we buy more accurate sensors of same type?
- Should we replace current sensors with other sensors that measure "different" variables?
- Should we buy additional sensors?

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- Should we buy better (less noisy) actuators?
- What is tradeoff between more accurate sensors and actuators?
- How do system dynamics effect such choices?

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REMARKS

- For linear systems such questions can be answered in a quantitative manner without doing any Monte-Carlo simulations.
- The error covariance equation is the main tool for such sensor-actuator accuracy tradeoffs.

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STATE DYNAMICS:

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t) + \underline{\xi}(t) \quad (1)$$

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) + \xi_2(t) \\ x_3(t) &= -x_2(t) + \xi_3(t) \end{aligned}\right\} \quad (2)$$

$$\rightarrow \underline{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad \underline{B} = \underline{0} \quad (3)$$

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SENSOR A

$$z(t) = x_1(t) + \theta(t) \qquad \rightarrow \underline{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (4)$$

SENSOR B

$$z(t) = x_1(t) + x_2(t) + \theta(t) \qquad \rightarrow \underline{C} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \quad (5)$$

SENSOR C

$$z(t) = x_1(t) + x_2(t) + x_3(t) + \theta(t) \qquad \rightarrow \underline{C} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad (6)$$

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PROBABILISTIC INFORMATION

- These parameters do not change with alternate sensor selector

- INITIAL STATE COVARIANCE: $\underline{\Sigma}_0$

$$\text{cov}\left[\underline{x}(t);\ \underline{x}(0)\right] = \underline{\Sigma}_0 \qquad (7)$$

$$= \begin{bmatrix} 40 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

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- PLANT NOISE COVARIANCE: $\underline{\Xi}\,\delta(t-\tau)$

$$\text{cov}\left[\underline{\xi}(t);\underline{\xi}(\tau)\right] = \underline{\Xi}\,\delta(t-\tau) \qquad (8)$$

$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}}_{\underline{\Xi}} \delta(t-\tau)$$

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- MEASUREMENT NOISE COVARIANCE: $\underline{\Theta}\,\delta(t-\tau)$

$$\text{cov}\left[\underline{\theta}(t);\underline{\theta}(\tau)\right] = \underline{\Theta}\,\delta(t-\tau) \qquad (9)$$

$$= \underbrace{1}_{\Theta = \text{ scalar}}\ \delta(t-\tau)$$

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• For each sensor, characterized by a different matrix $\underline{C}$, one computes the 3x3 error covariance matrix $\underline{\Sigma}(t)$ by forward in time integration of the matrix Riccati equation.

$$\dot{\underline{\Sigma}}(t) = \underline{A}\,\underline{\Sigma}(t) + \underline{\Sigma}(t)\underline{A}' + \underline{\Xi} - \underline{\Sigma}(t)\,\underline{C}'\underline{\Theta}^{-1}\,\underline{C}\,\underline{\Sigma}(t);\ \underline{\Sigma}(0) = \underline{\Sigma}_0 \qquad (10)$$

and then the 3x1 filter gain $\underline{H}(t)$

$$\underline{H}(t) = \underline{\Sigma}(t)\,\underline{C}'\underline{\Theta}^{-1} = \begin{bmatrix} h_1(t) \\ h_2(t) \\ h_3(t) \end{bmatrix} \qquad (11)$$

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• FOLLOWING FIGURES SHOW

1. Complete structure of Kalman filter for each sensor
2. The three time-varying filter gains $h_1(t)$, $h_2(t)$, $h_3(t)$
3. The three estimation error variances $\sigma_{11}(t)$, $\sigma_{22}(t)$, $\sigma_{33}(t)$ -- i.e., diagonal elements of $\underline{\Sigma}(t)$.

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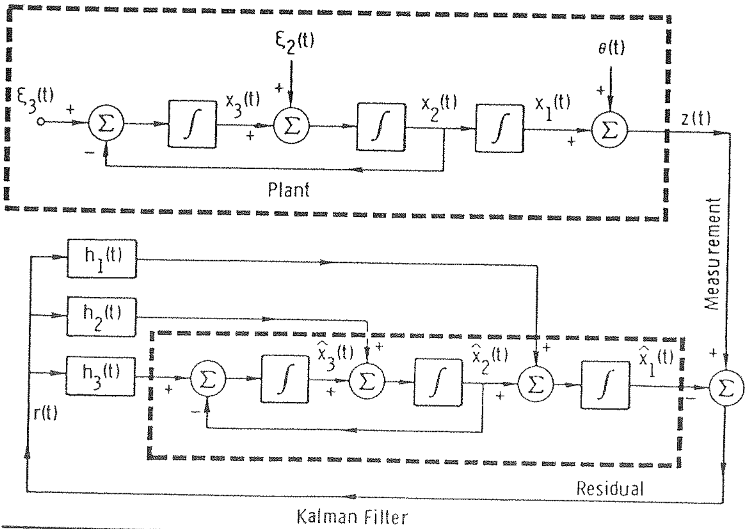
STRUCTURE FOR SENSOR A



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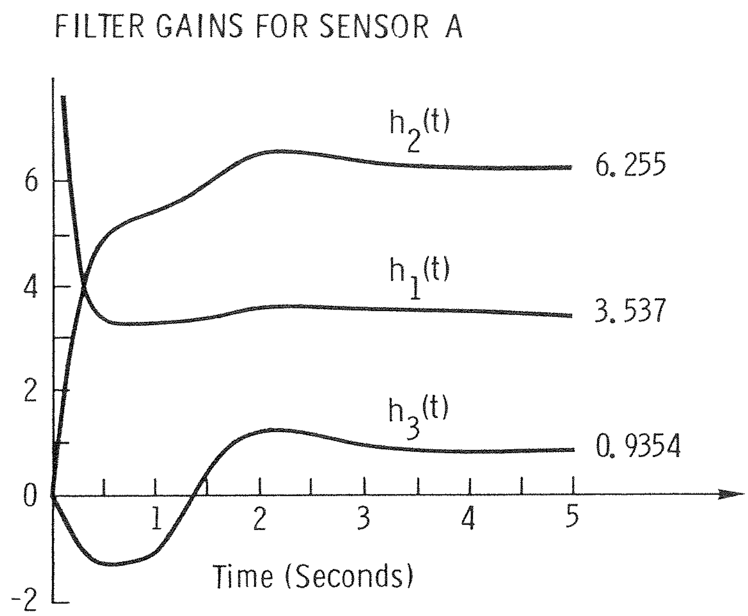
FILTER GAINS FOR SENSOR A
h2(t)
6.255
h1(t)
3.537
h3(t)
0.9354
6
4
2
0
-2
1
2
3
4
5
Time (Seconds)

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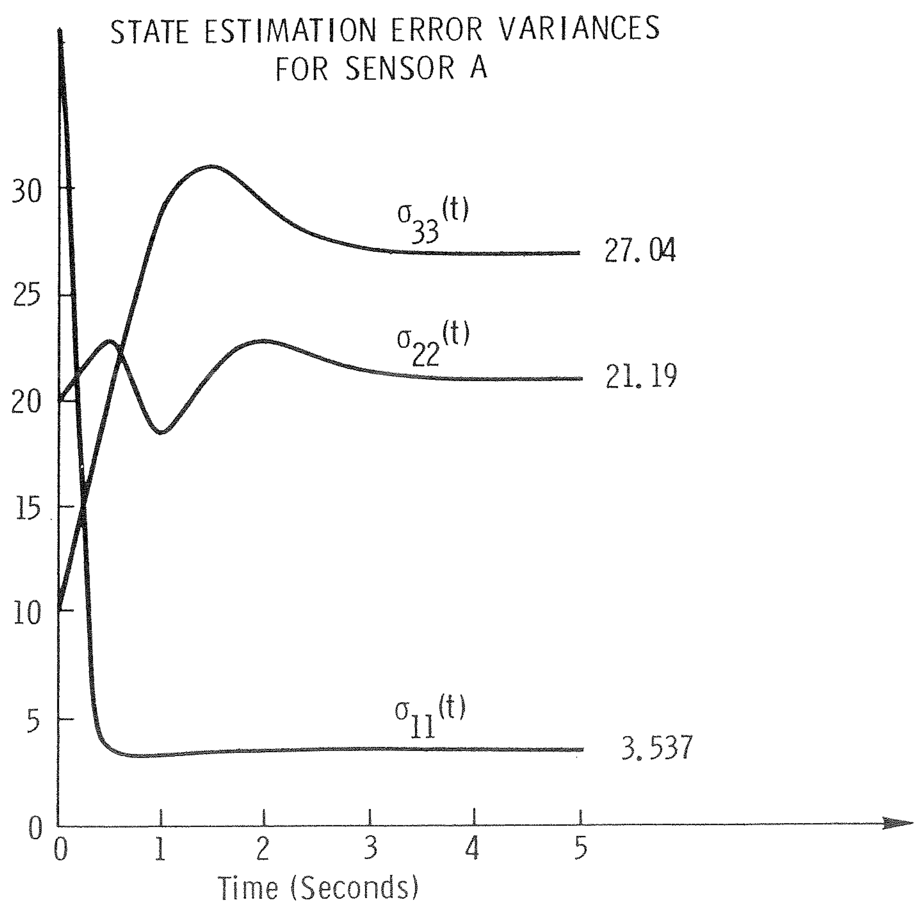
STATE ESTIMATION ERROR VARIANCES
FOR SENSOR A
σ33(t)
27.04
σ22(t)
21.19
σ11(t)
3.537
30
25
20
15
10
5
0
0
1
2
3
4
5
Time (Seconds)

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CONJECTURE

- To reduce velocity and acceleration errors, buy a sensor (sensor B) that measures linear combination of position and velocity
- Sensor B

  $z(t) = x_1(t) + x_2(t) + \theta(t)$

contains measurement "closer" to white noise sources

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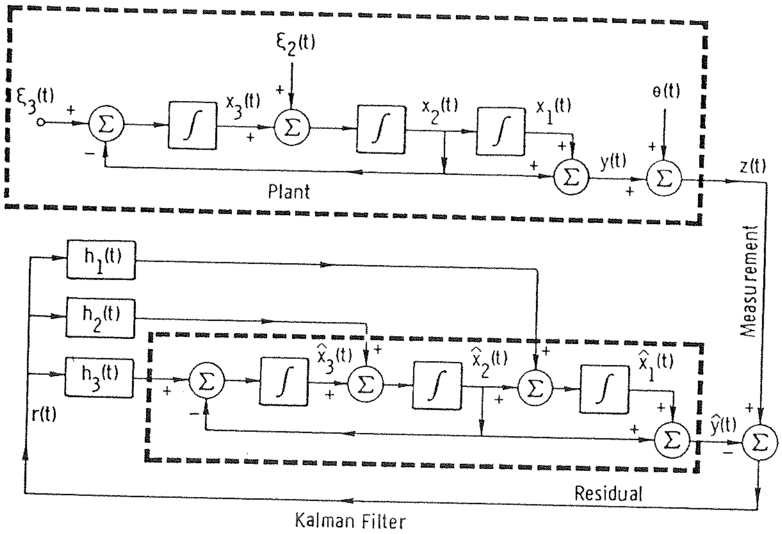
STRUCTURE FOR SENSOR B



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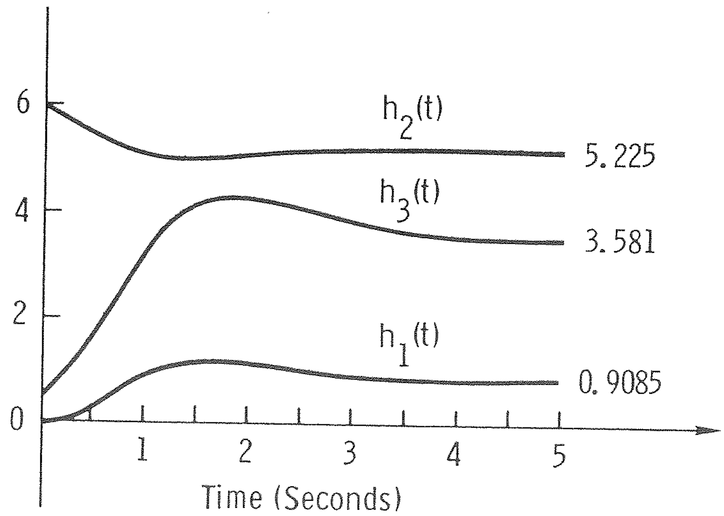
FILTER GAINS FOR SENSOR B



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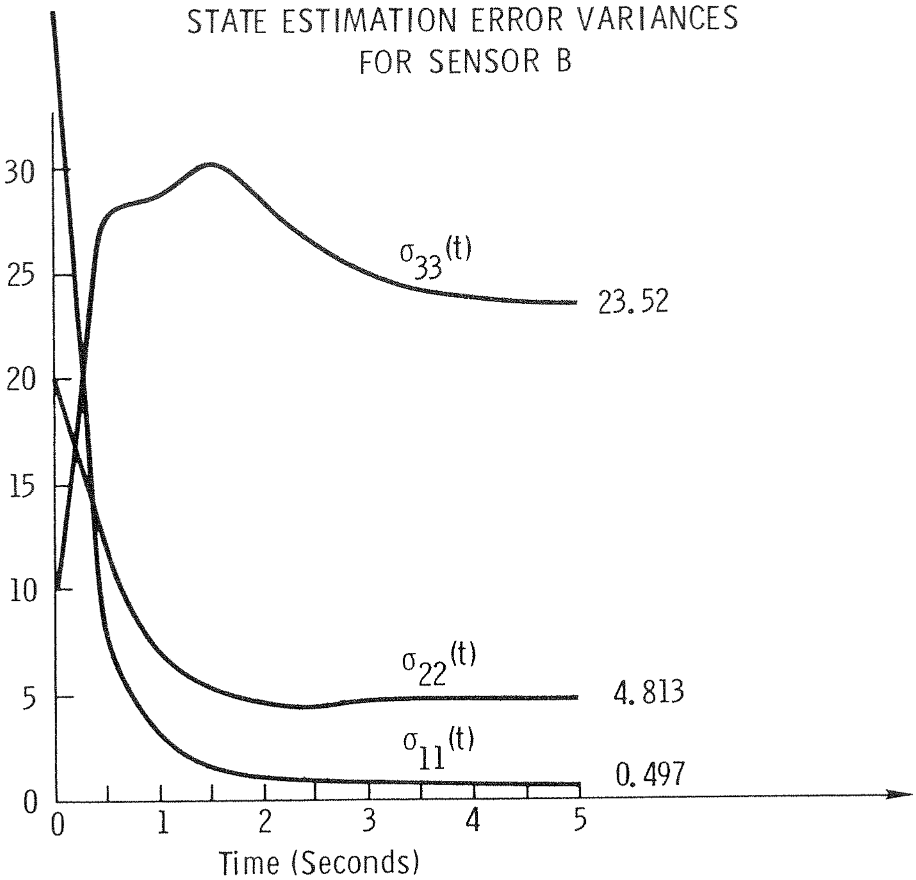
STATE ESTIMATION ERROR VARIANCES FOR SENSOR B



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DISCUSSION

- Idea of having sensor measure signals closer to white noise "worked"

  Sensor B is better than Sensor A

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- To further reduce uncertainty in acceleration measurement device Sensor C that measures a linear combination of position, velocity, acceleration
- Sensor C

  $z(t) = x_1(t) + x_2(t) + x_3(t) + \theta(t)$

- We may expect to find

  Sensor C is better than Sensor B (?)

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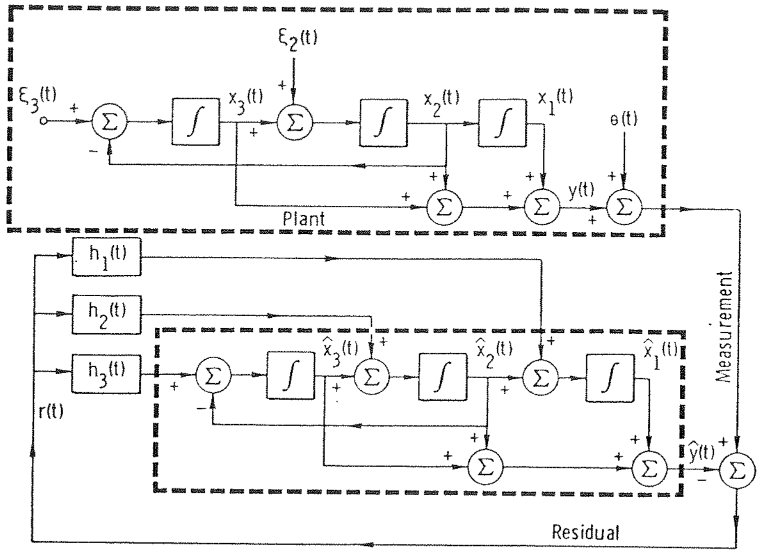
STRUCTURE FOR SENSOR C



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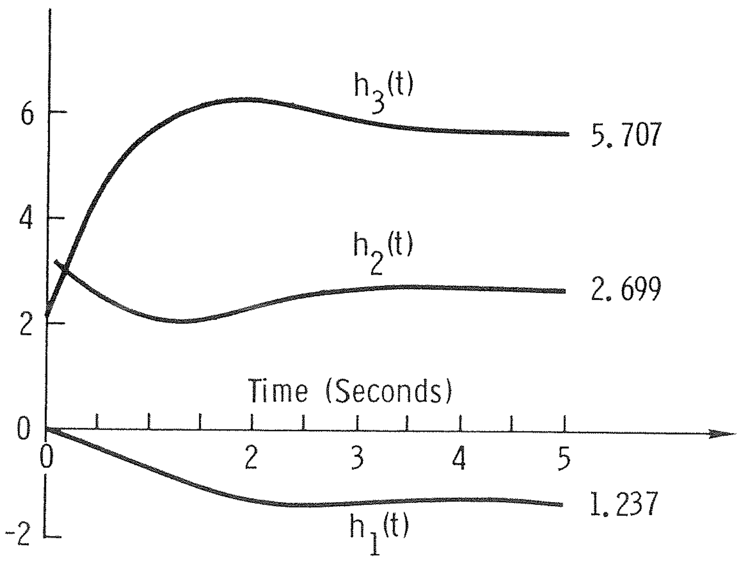
FILTER GAINS FOR SENSOR C



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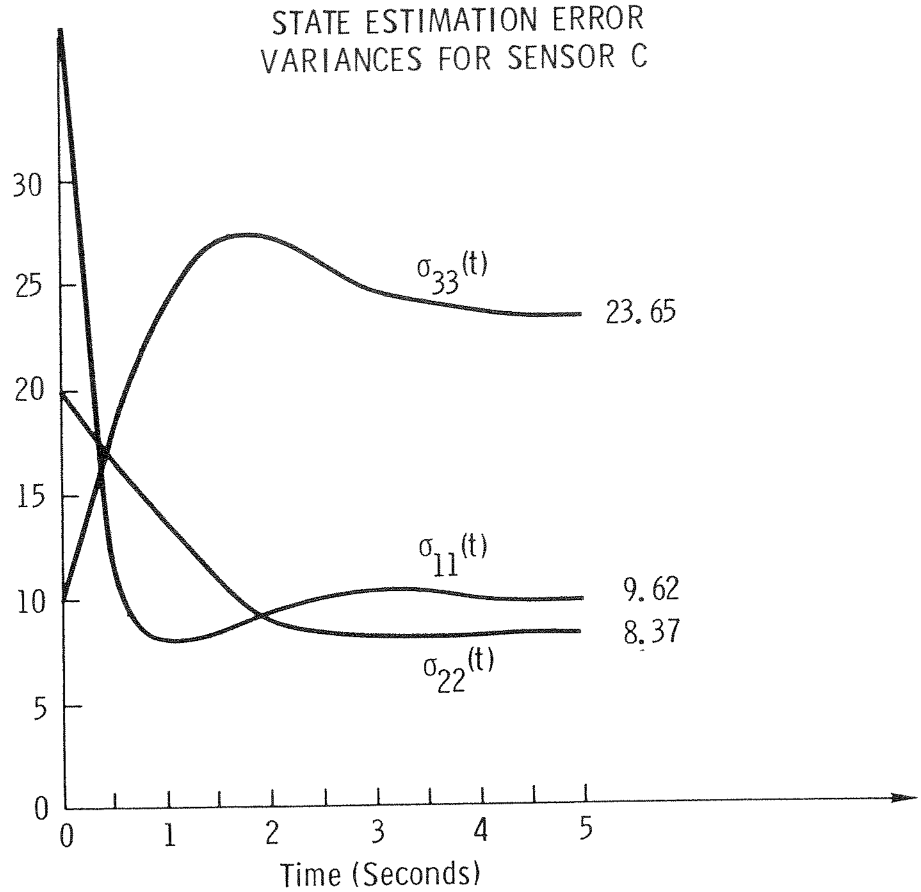

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ACCURACY TABLE

Steady State Variances	Sensor A	Sensor B (Best)	Sensor C
σ_{11}	3.537	0.497	9.62
σ_{22}	21.19	4.813	8.37
σ_{33}	27.04	23.52	23.65

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LESSON

Intuitive arguments may not be true if internal feedback in system dynamics is not taken into account.

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EFFECTS OF DECREASE IN PLANT NOISE INTENSITIES USING SENSOR C

- Original plant noise intensity matrix

$$\underline{\Xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad (12)$$

- New plant noise intensity matrix

$$\underline{\Xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad (13)$$

  - Much more accurate "actuators"

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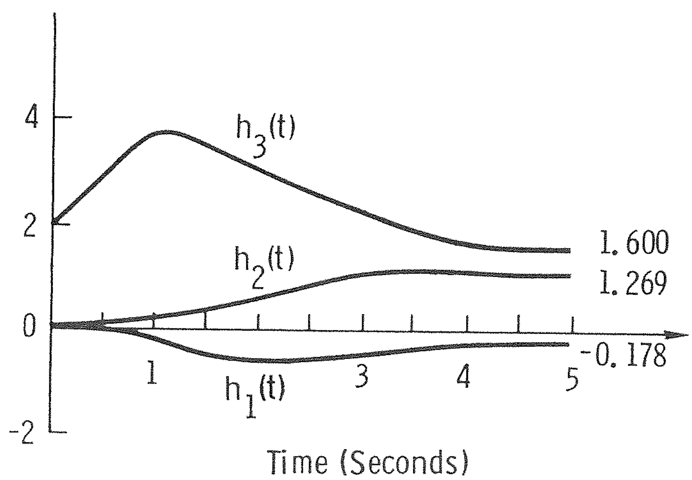
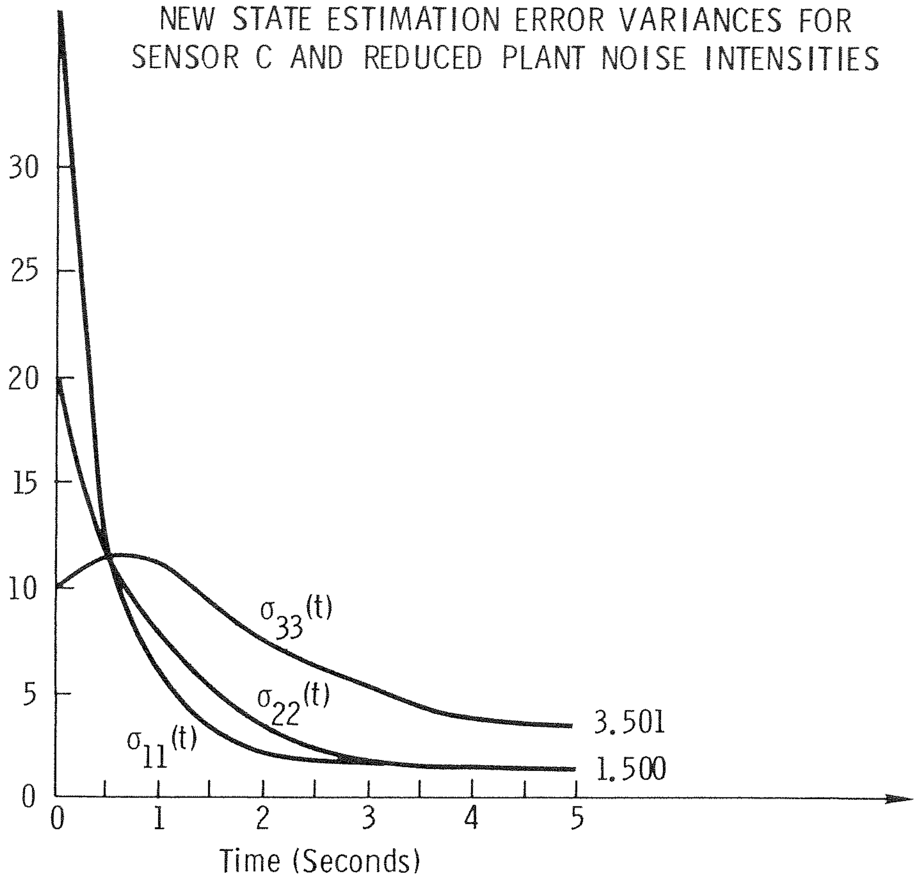
NEW FILTER GAINS FOR SENSOR C WITH REDUCED PLANT NOISE INTENSITIES




NEW STATE ESTIMATION ERROR VARIANCES FOR
SENSOR C AND REDUCED PLANT NOISE INTENSITIES
30
25
20
15
10
5
0
σ33(t)
σ22(t)
σ11(t)
3.501
1.500
0
1
2
3
4
5
Time (Seconds)