1

STOCHASTIC ESTIMATION

Numerical example: Estimation of position, velocity, and ballistic parameter for a vertical re-entering body

2

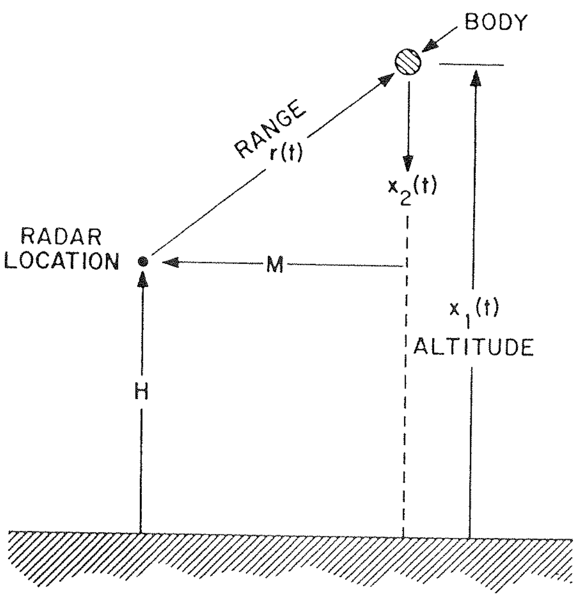
MOTIVATION

1) To illustrate differences between

- extended Kalman filter (EKF) (or first-order filter)
- second-order filter (SOF) (or gaussian filter)

2) To demonstrate that biases that build up in first order filter can be removed partially by the second-order filter

3



Problem Geometry

4

VARIABLE DEFINITIONS

$x_1(t)$: altitude
$x_2(t)$: velocity (downward)
m: mass (constant)
C_D: drag coefficient (constant)
A: reference area for drag evaluation (constant)
ρ: mass density of the atmosphere
H: radar altitude
M: horizontal distance = 100, 000 ft
$r(t)$: true range

5

DIFFERENTIAL EQUATIONS

• Neglect force of gravity

$$\left.\begin{aligned} \dot{x}_1(t) &= -x_2(t) \\ \dot{x}_2(t) &= -\frac{C_D A \rho}{2m}\, x_2^2(t) \end{aligned}\right\} \qquad (1)$$

• Air density

$$\left.\begin{aligned} \rho &= \rho_0 \exp\left(-\gamma x_1(t)\right) \\ \gamma &= 5 \times 10^{-5} \end{aligned}\right\} \qquad (2)$$

6

• Define new state variable

$$x_3(t) \triangleq C_D A\, \rho_0 / 2m = \text{constant} \qquad (3)$$

x_3 "small" $\Rightarrow$ heavy body
x_3 "large" $\Rightarrow$ light body

• Final differential equations

$$\left.\begin{aligned} \dot{x}_1(t) &= -x_2(t) \\ \dot{x}_2(t) &= -\exp\left(-\gamma x_1(t)\right) x_2^2(t)\; x_3(t) \\ \dot{x}_3(t) &= 0 \end{aligned}\right\} \qquad (4)$$

7
MEASUREMENTS

- Only range is measured at discrete time instants (once a second)

$$z(t) = \sqrt{M^2 + [x_1(t) - H]^2} + \theta(t) \qquad (5)$$

$t = 1, 2, 3, \ldots$

- $\theta(t)$ discrete zero mean noise with covariance

$\Theta = 10{,}000$ (100 ft RMS error)

8
INITIAL COVARIANCE MATRIX

$$\underline{\Sigma}(0) = \begin{bmatrix} 10^6 & 0 & 0 \\ 0 & 4 \times 10^6 & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix} \qquad (6)$$

9
INITIAL STATE

- True

$x_1(0) = 300{,}000$ ft

$x_2(0) = 20{,}000$ ft/sec

$x_3(0) = 10^{-3}$ (light target)

- Estimate

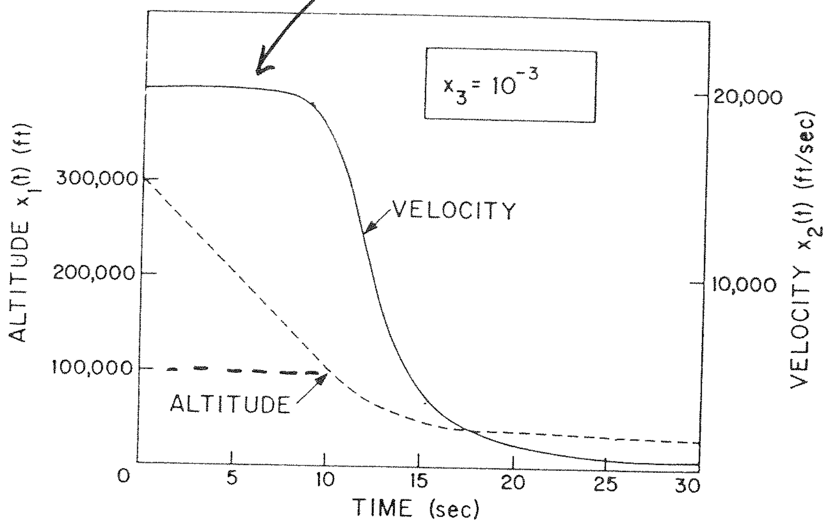
$\hat{x}_1(0) = 300{,}000$ ft

$\hat{x}_2(0) = 20{,}000$ ft/sec

$\hat{x}_3(0) = 3 \times 10^{-5}$ (heavy target)

10

Negligible atmosphere ⇒ dynamics almost linear



True Target Trajectory

11

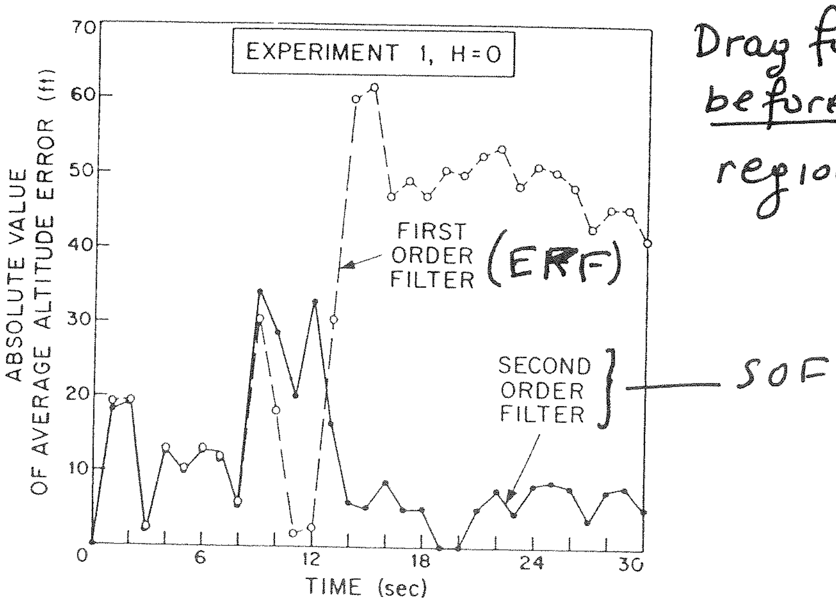
EXPERIMENTS (we shall show results based on averages of 50 Monte Carlo runs)

#1: H = 0 ft
Radar on ground

#2: H = 100,000 ft
Radar "up"

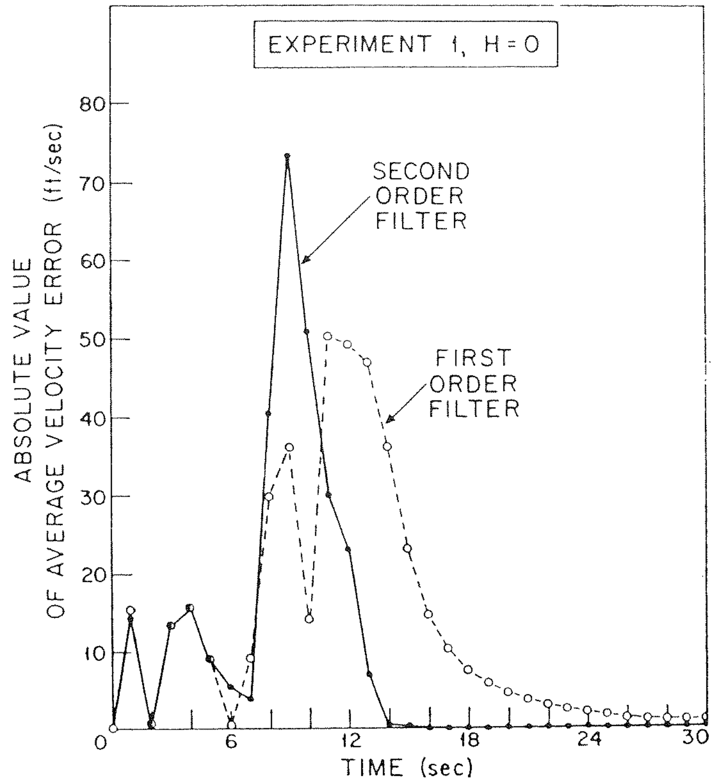
#3: H = 200,000 ft
Radar "higher"

12

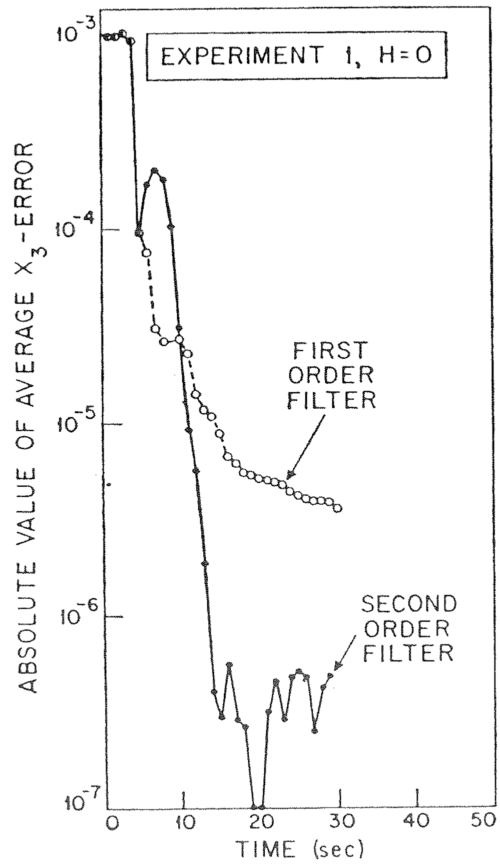


Drag forces build-up before poor observability region

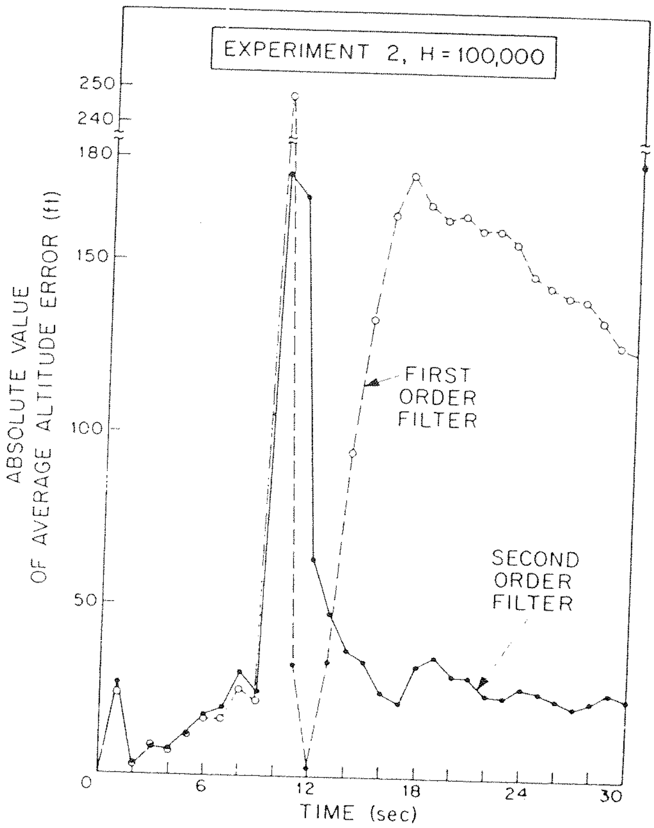
13


EXPERIMENT 1, H=0
SECOND ORDER FILTER
FIRST ORDER FILTER
ABSOLUTE VALUE OF AVERAGE VELOCITY ERROR (ft/sec)
TIME (sec)
0
6
12
18
24
30
10
20
30
40
50
60
70
80

14

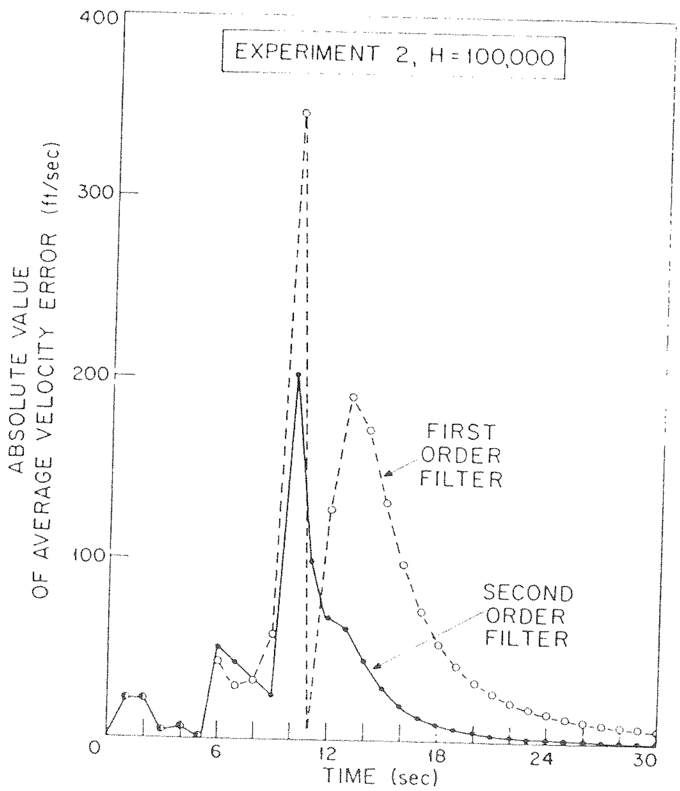

EXPERIMENT 1, H=0
FIRST ORDER FILTER
SECOND ORDER FILTER
ABSOLUTE VALUE OF AVERAGE X_3-ERROR
TIME (sec)
0
10
20
30
40
50
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}

15

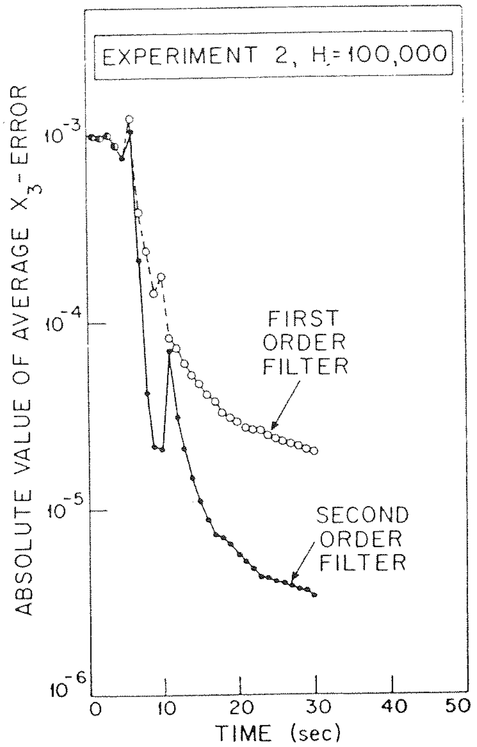


Target is most "unobservable" when nonlinear drag dominates

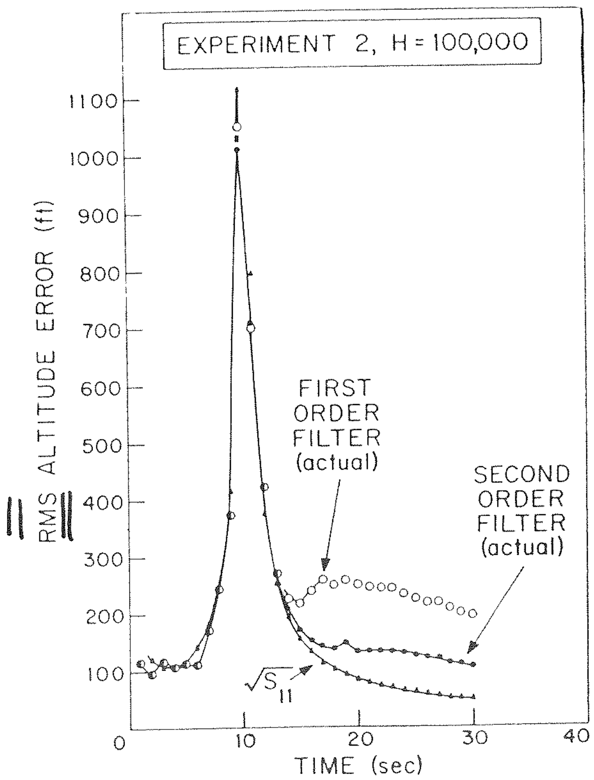
16



17

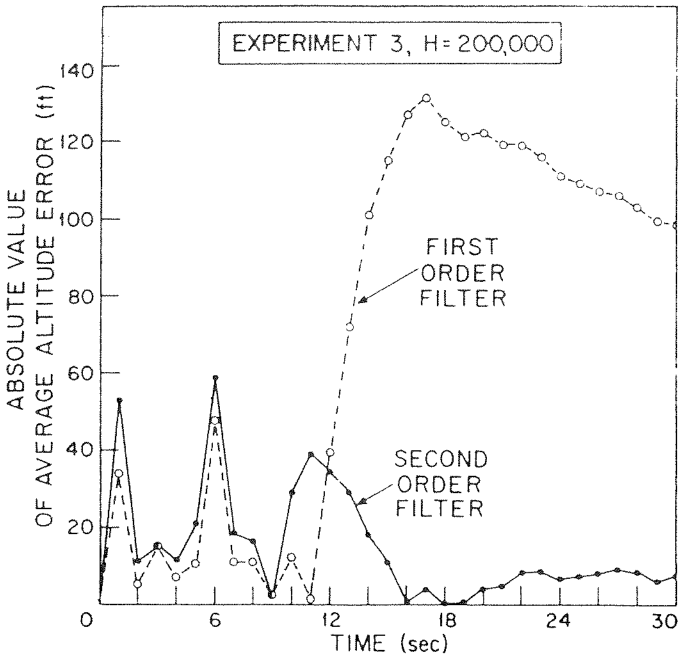


18



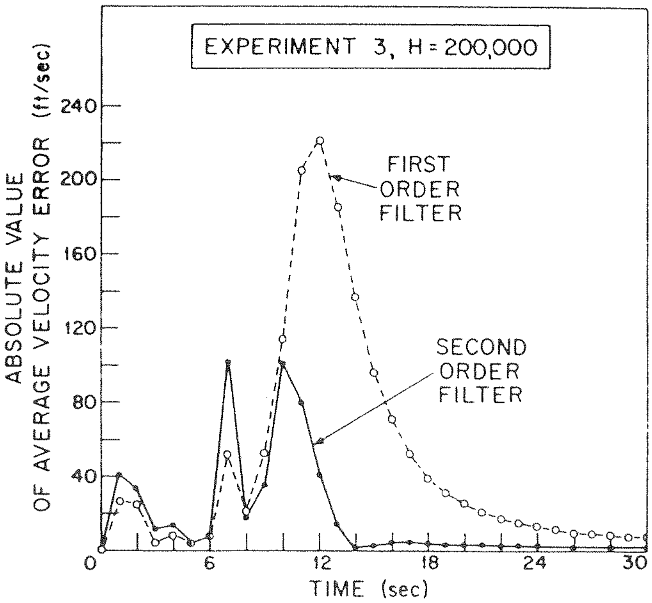
- $\sqrt{S_{11}}$ is calculated by EKF.
- EKF "knows" poor observability near 9-10 secs
- EKF is more "optimistic" after ~ 12 secs
- Should use fake white noise to "raise" $\sqrt{S_{11}}$

19

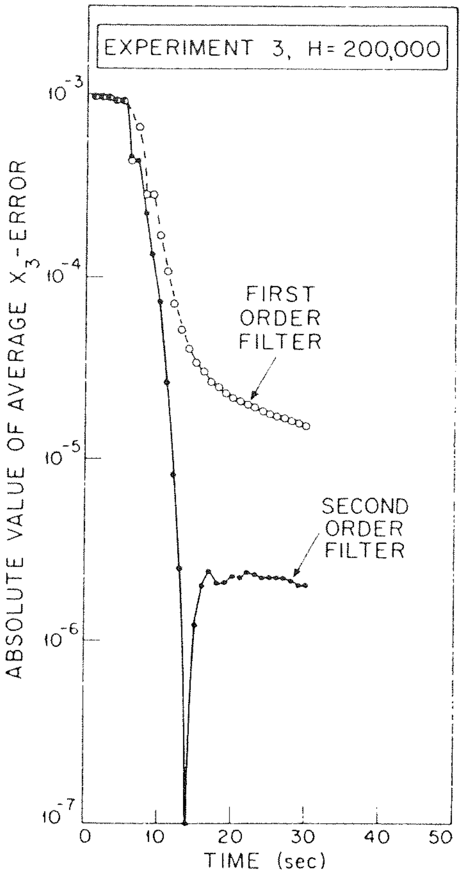


Poor observability region (4–5 secs) is before time of significant drag nonlinearity

20



21



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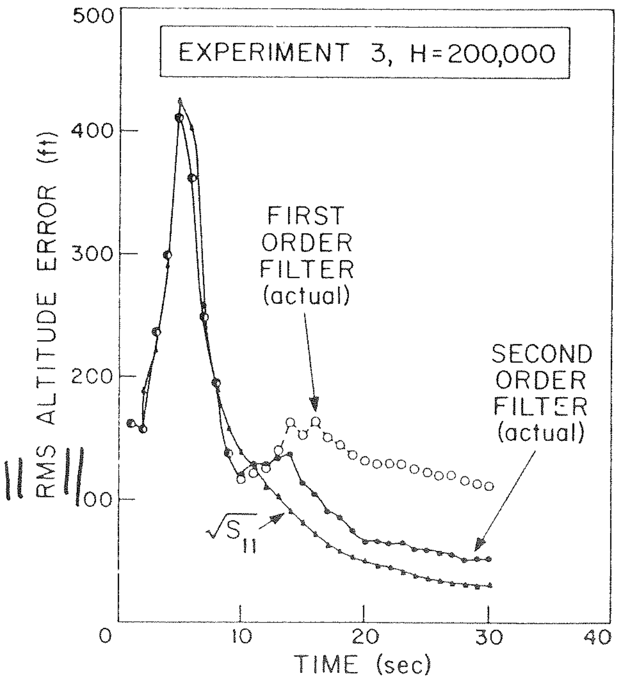

- EKF "knows" poor observability (~ 4-5 secs).
- Again, calculated EKF variance too optimistic
- Should "raise" by adding fake white noise

23

FINAL REMARKS

- Second order filter at least twice as good as extended Kalman filter
- Second order filter requires only about 15% more computation time as compared to the extended Kalman filter

← For this example!