Introduction to Optimal Estimation

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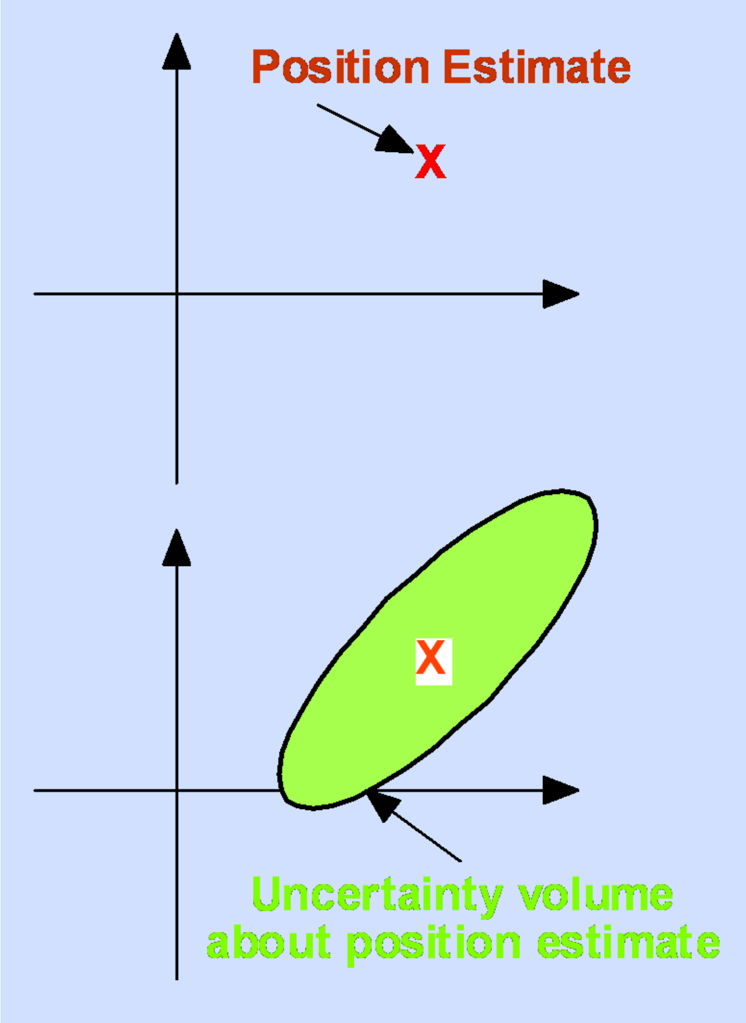
Ref. KF #1

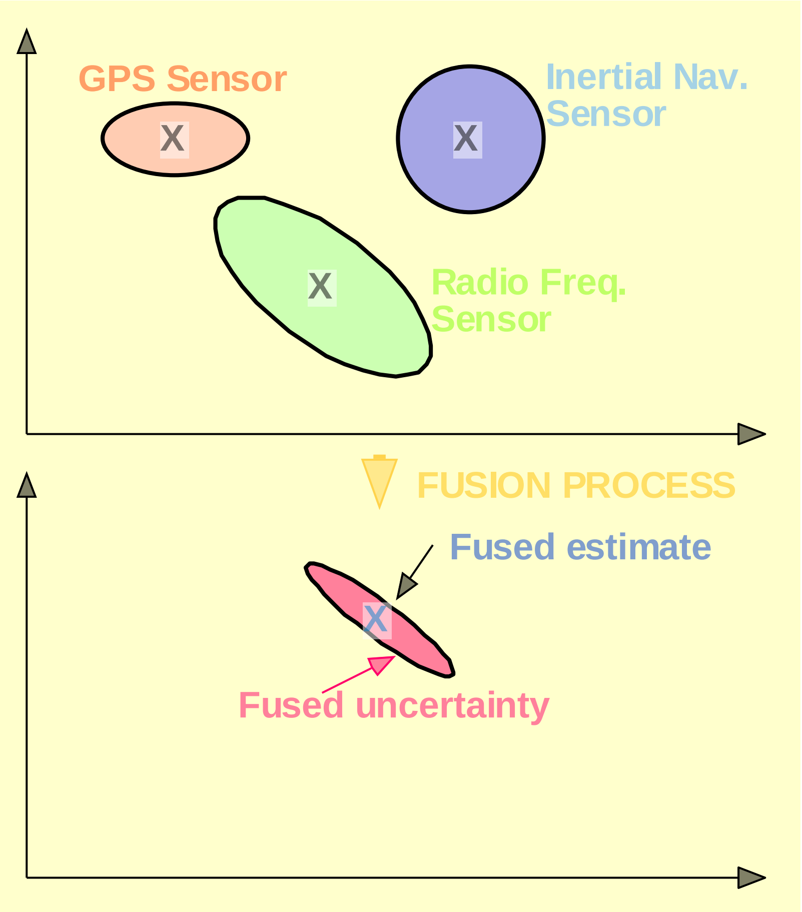
The Basic Estimation Questions

- Where am I?
- How much do I believe the “where am I” estimate



The Concept of Sensor Fusion

- Different sensor types yield different position estimates (and uncertainty volumes)
- SENSOR FUSION combines the measurements from all the different sensors to yield
  - "better" updated position estimate
  - reduced volume of uncertainty about fused estimate
- Sensor fusion is a centralized decision problem



Never Forget ...

- Obtaining only an estimate of a quantity is never enough
- Being “right” on the average is nice, but not enough
- We must also obtain a measure of the quality (or believability) of the estimate
- We need something like a standard-deviation or some bounding measure
- Example. An estimate of, say, 5.2 with ±23% uncertainty is “worse” than an estimate of, say, 5.12 with ±7% uncertainty

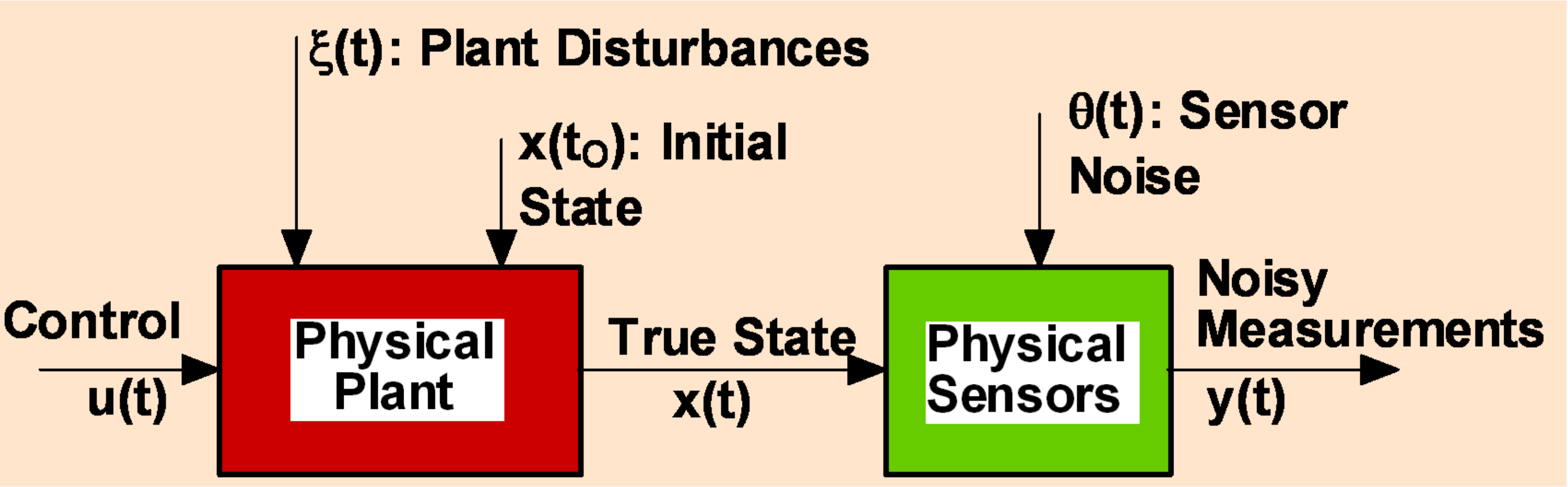
Theme

- We shall deal with the dynamic evolution of uncertainty, so we must combine
  - stochastic processes (time-varying random variables)
  - linear and nonlinear dynamic systems, whose state-evolution depends on stochastic processes
- We shall define the three common classes of estimation problems
  - filtering
  - prediction (or forecasting)
  - smoothing (or interpolation)
- We shall employ "optimal" methods for extracting information from uncertain measurements, and avoid ad-hoc processes
- Expert understanding of the filtering problem is essential for the solution of the prediction and/or the smoothing problem

Dynamic Systems and "Where am I?"

- Recall that the state variable description of dynamical systems is useful because
  - knowledge of the present state summarizes past behavior
  - knowing the present state and future inputs is sufficient to determine exactly all future states and output (sensor) variables
- The question "where am I?" in dynamic systems (plants) corresponds to the estimation of the entire state vector
  - positions, velocities, accelerations ... in mechanical systems
  - Pressures, temperatures ... in thermodynamic systems
  - inductor currents, capacitor voltages ... in linear electric circuits
  - We shall deal with state estimation problems where the plant is subject to stochastic disturbances, and on the basis of noisy sensor measurements

The Physical System



- The only real-time signals that are available are the control(s) and the noisy sensor measurement(s)
- Cannot directly measure the actual state variables or plant disturbances or sensor noise

Example: A Sailboat

STATE VARIABLES

- 3D positions and velocities
- Roll, yaw, and pitch angles
- Roll, yaw, and pitch rates

CONTROL VARIABLES

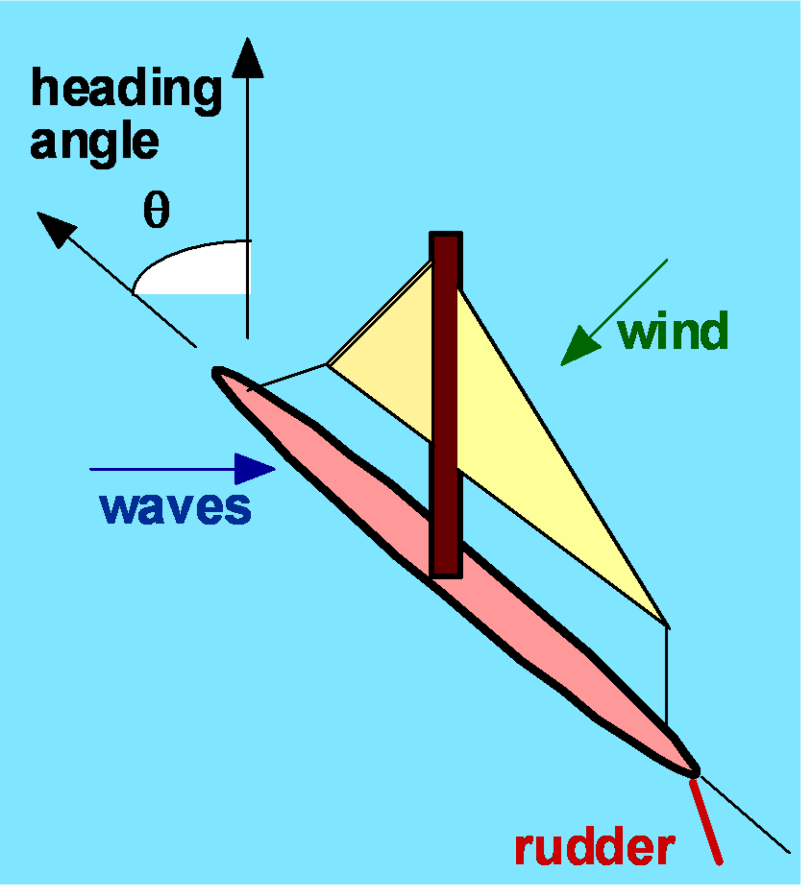
- Rudder angle
- Sail area and angle

PLANT DISTURBANCES

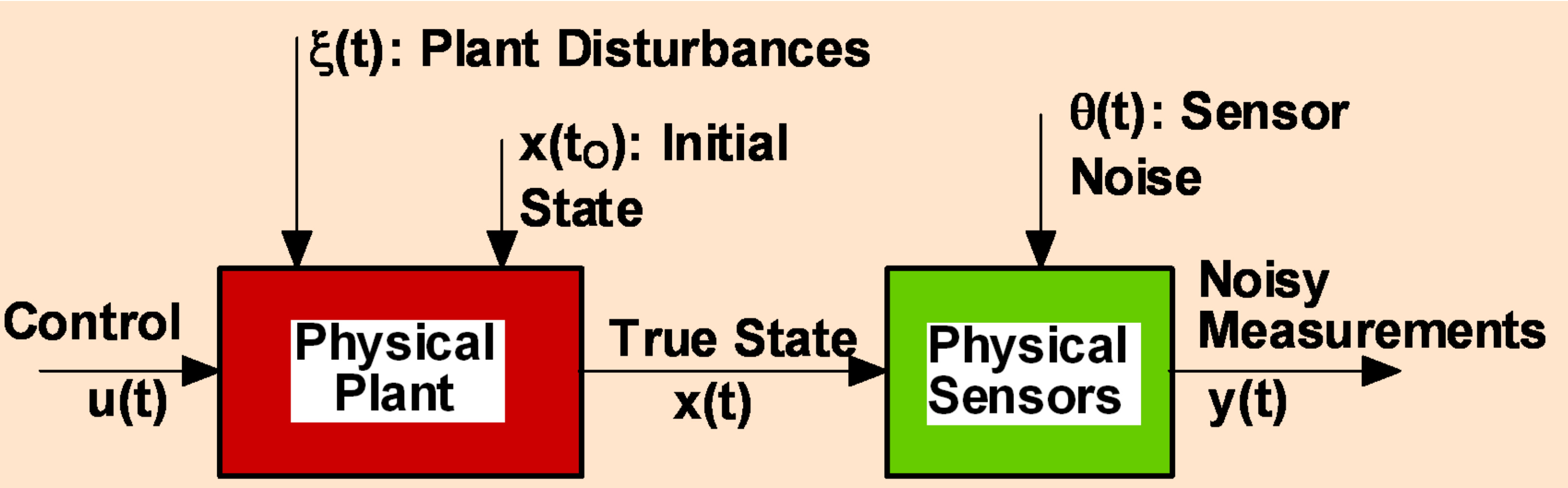
- Wind and wave forces and moments (including fluctuations from mean)

NOISY MEASUREMENTS

- Heading angle, yaw angle and rate(?), roll angle and rate(?), ...



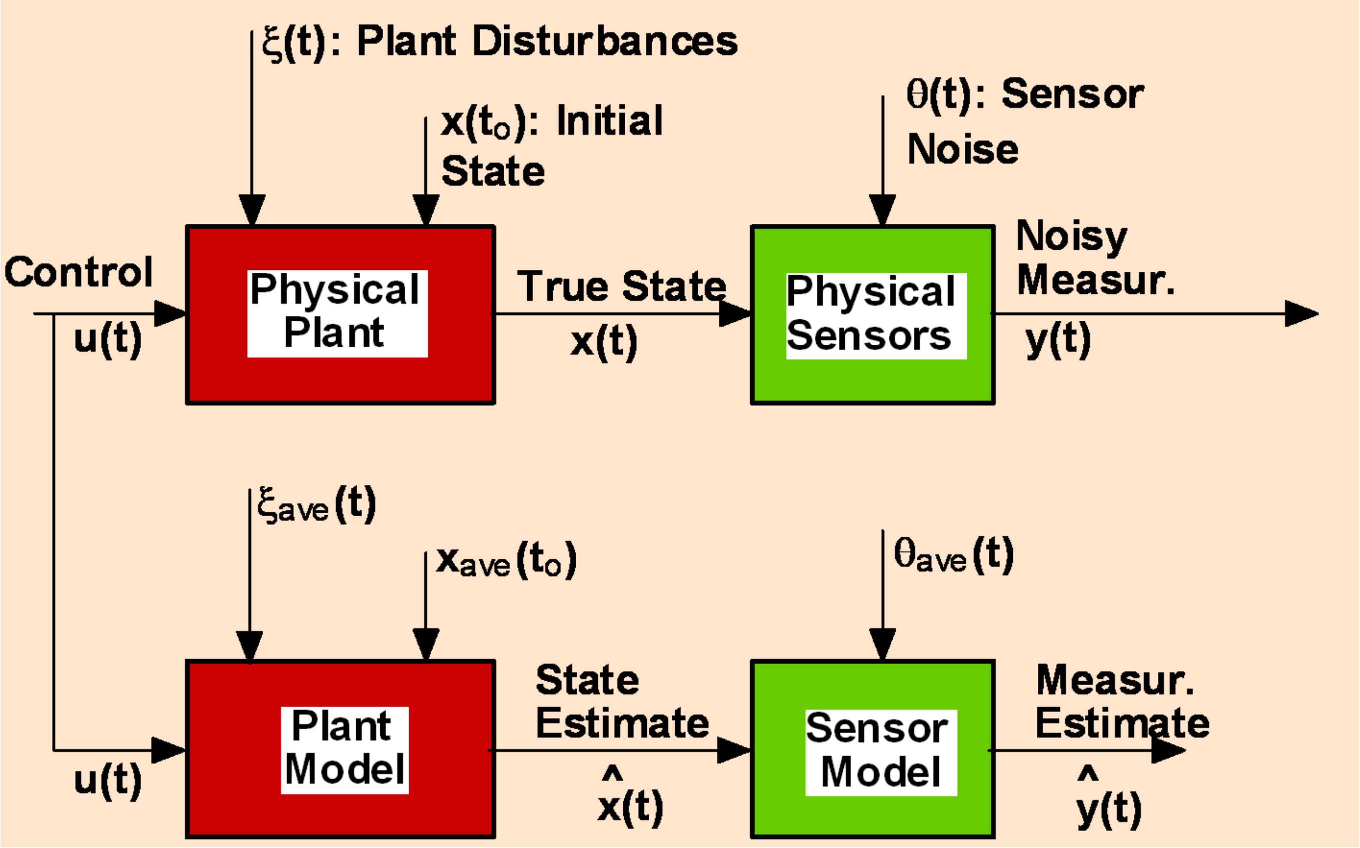
The Physical System: Uncertainties



- Exogenous uncertainties
  - initial state is random vector
  - plant disturbance is vector-valued random process (or sequence)
  - sensor noise is vector-valued random process (or sequence)
- Mathematical models of plant dynamics and sensors are inaccurate

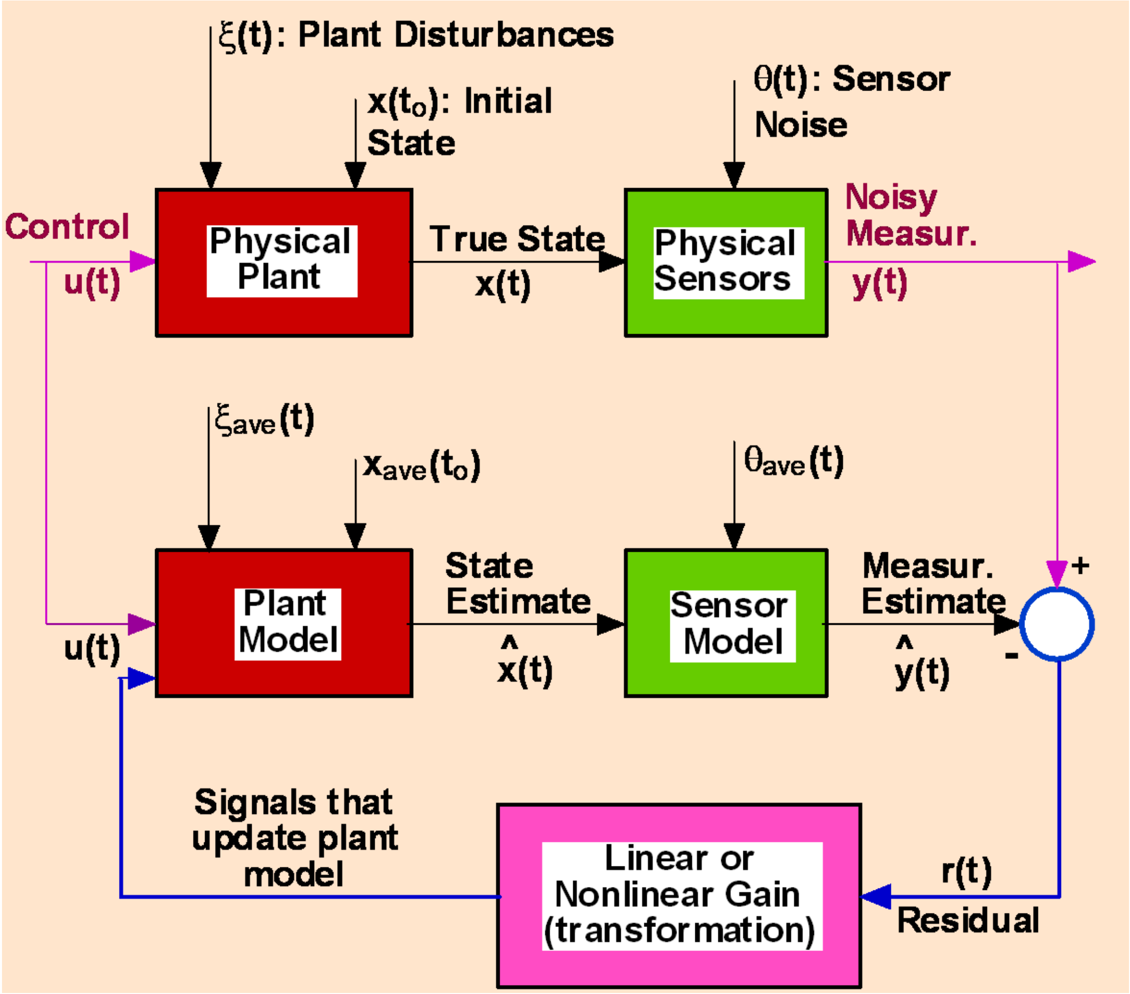
Open-Loop Prediction

- Make a mathematical model of the plant and sensors, driven by known control
- Use average values of initial state, plant disturbance and sensor noise

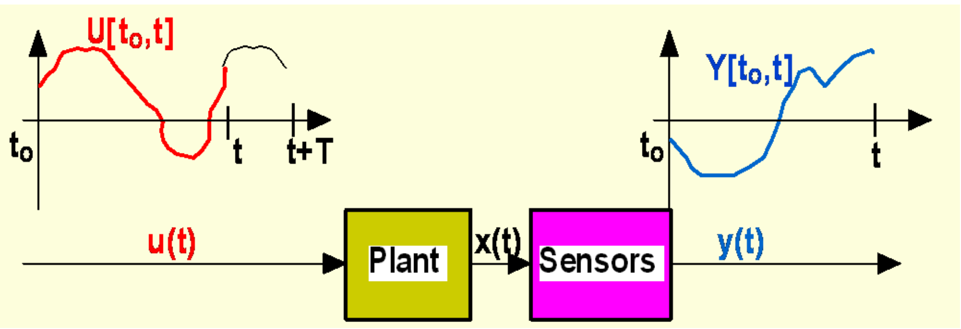


Estimator (Filter) Structure

- Use real-time information to provide input signals to the plant model, so as to improve state estimates



Filtering, Prediction and Smoothing



FILTERING PROBLEM

- Given input time function $U[t_0, t]$ and measurement time function $Y[t_0, t]$, find "best" estimate of the state $x(t)$

PREDICTION PROBLEM

- Let T be a prediction time.
- Given $U(t_0, t+T)$ and $Y(t_0, t)$
- Determine "best" estimate of (future) state $x(t+T)$

SMOOTHING PROBLEM

- Let τ be any time, $t_0 \leq \tau \leq t$
- Given $U(t_0, t)$ and $Y(t_0, t)$
- Determine "best" estimate of (past) state $x(\tau)$

The Nature of Mathematical Models

- Dynamic models of the physical plant, with finite number of state variables
- Static models of sensor measurements

CONTINUOUS-TIME MODELS: $t_0 \leq t$

- State dynamics described by ordinary vector differential equations

$$\frac{d}{dt} x(t) = f\big(x(t), u(t), \xi(t), t\big)$$

$$y(t) = g(x(t), u(t), \theta(t), t)$$

DISCRETE-TIME MODELS: $t = 0, 1, 2, \ldots,$

- State dynamics described by vector difference equations

$$x(t+1) = f\big(x(t), u(t), \xi(t), t\big)$$

$$y(t) = g(x(t), u(t), \theta(t), t)$$

Linear Dynamical Systems

Linear Time-Varying (LTV) Systems

- Continuous-time

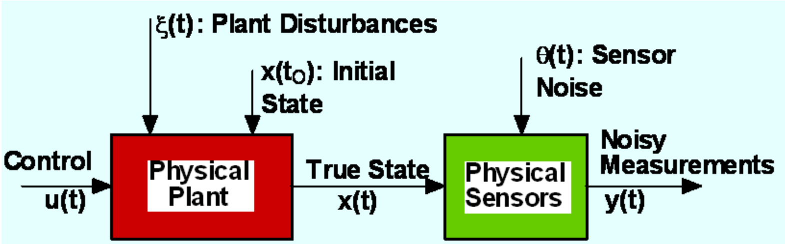
$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t) + L(t)\xi(t)$$

$$y(t) = C(t)x(t) + \theta(t)$$

- Discrete-time

$$x(t+1) = A(t)x(t) + B(t)u(t) + L(t)\xi(t)$$

$$y(t) = C(t)x(t) + \theta(t)$$



Linear Time-Invariant (LTI) Systems

- Continuous-time

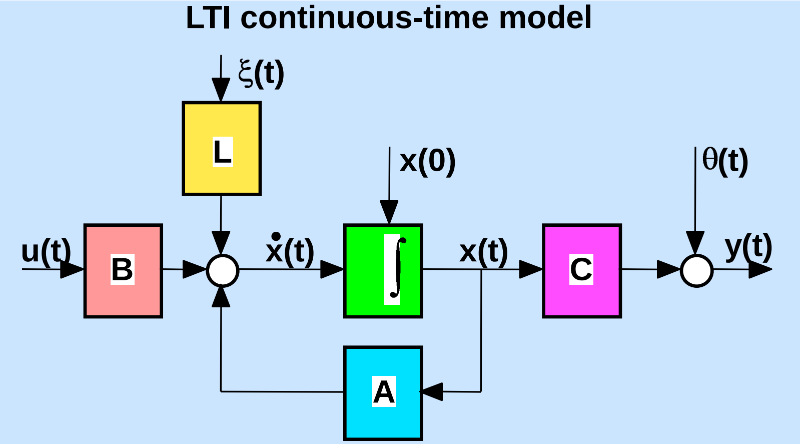
$$\frac{d}{dt}x(t) = Ax(t) + Bu(t) + L\xi(t)$$

$$y(t) = Cx(t) + \theta(t)$$

- Discrete-time

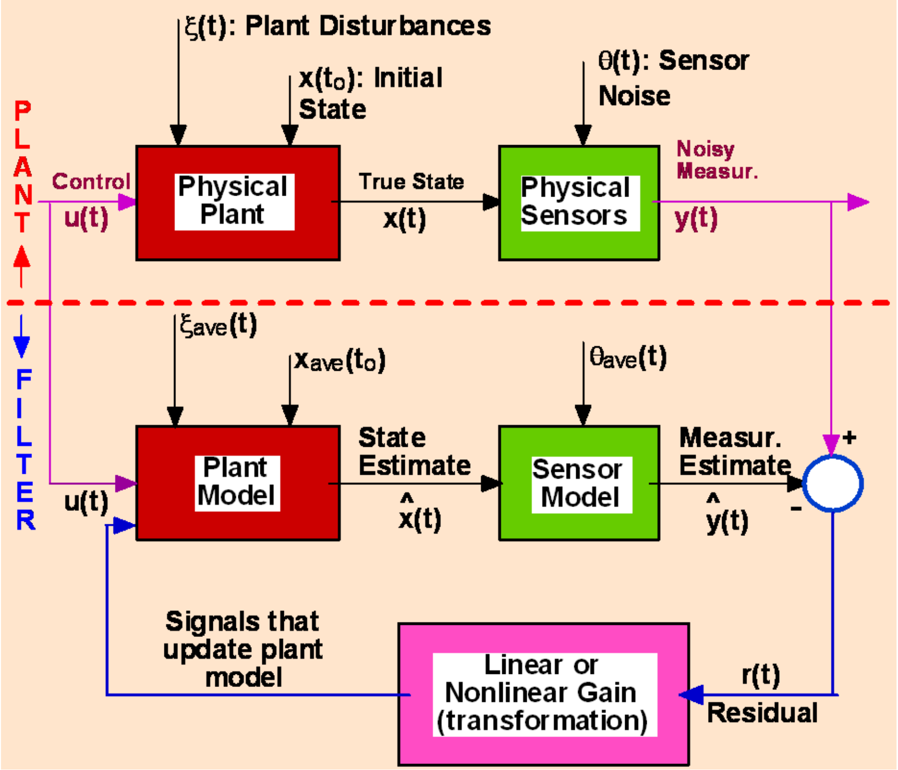
$$x(t+1) = Ax(t) + Bu(t) + L\xi(t)$$

$$y(t) = Cx(t) + \theta(t)$$



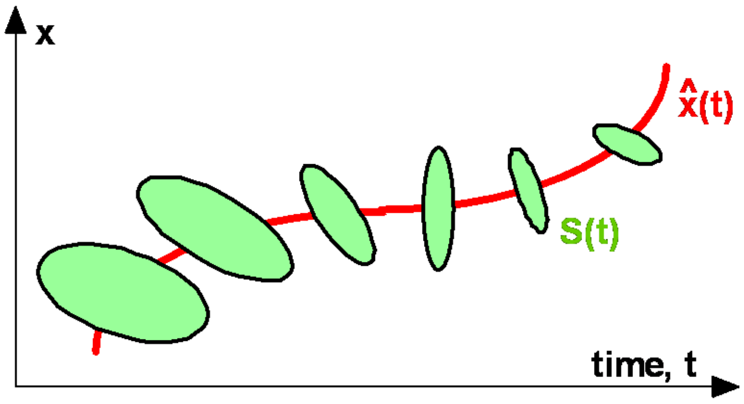
Computational Considerations

- The filter must operate in real-time
- We must solve in real-time the plant-model dynamical equations and sensor-model equations
- The types of the impleme-nted transformations (that map the residuals to plant model corrections) are dicta-ted by real-time computer constraints
  - optimal
  - suboptimal



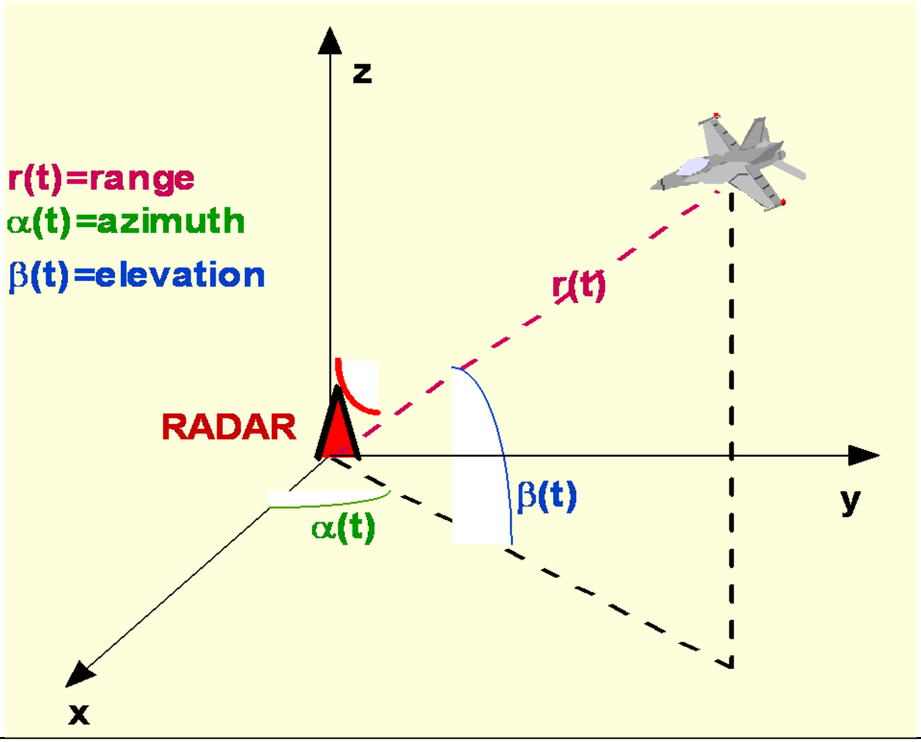
Dynamic Evolution of Uncertainty

- In dynamic estimation the state-vector estimate evolves with time
- The covariance matrix of the state estimation vector can be used to quantify the volume of uncertainty about the state estimate
- Thus we must find the dynamic equations that govern the dynamic evolution of BOTH the state estimate and error-covariance matrix



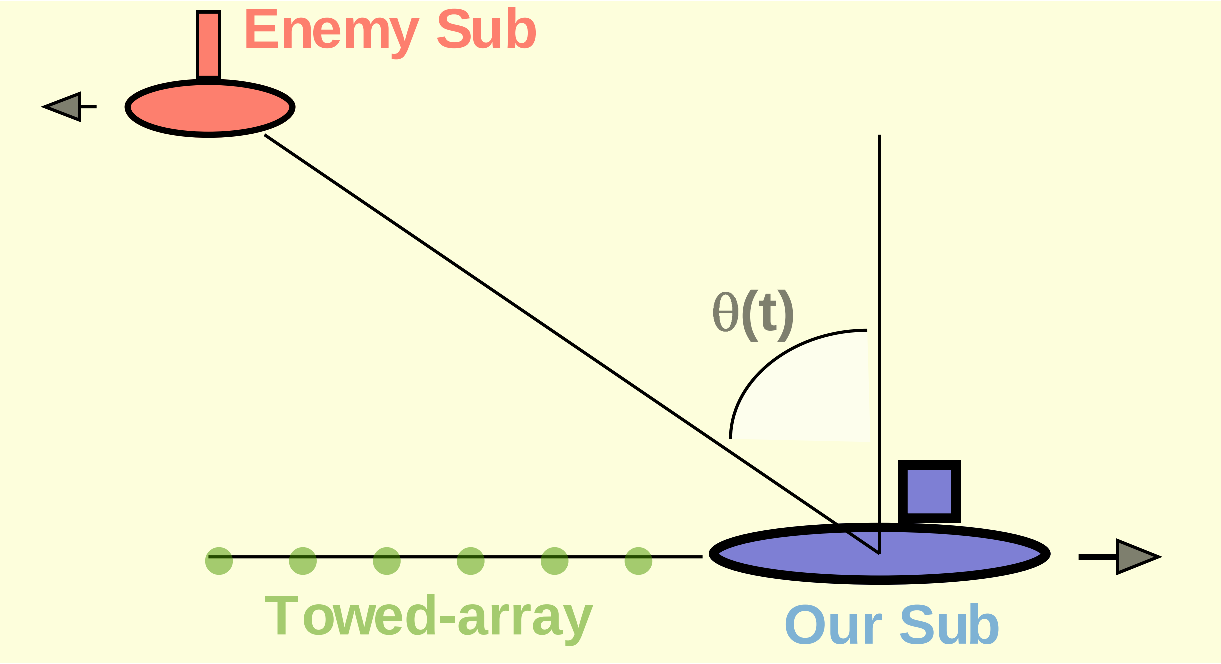
Radar (Ladar) Tracking

Estimate (3D) positions, velocities, and perhaps accelerations of moving aerospace target, based upon noisy measurements of range, azimuth and elevation



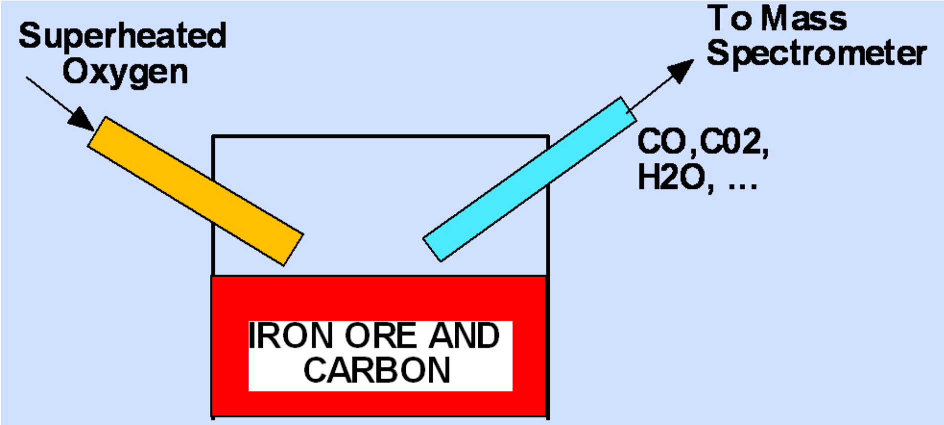
Passive Sonar Tracking

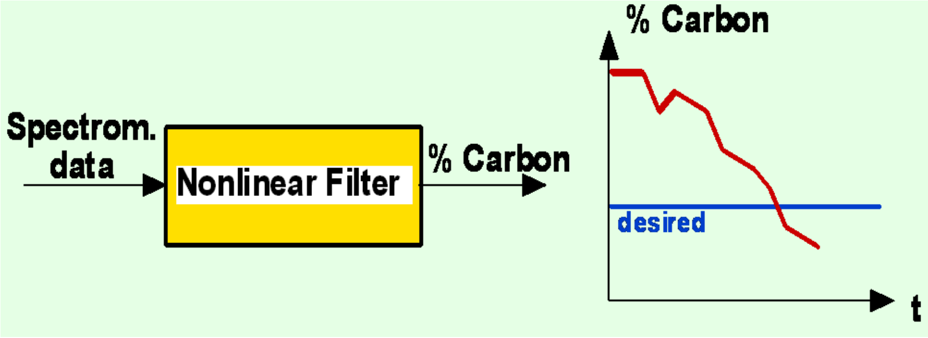
- Towed-array (an array of microphones) deployed behind our submarine can measure azimuth angle to enemy submarine
- No range measurement available, for stealth



Basic Oxygen Furnace

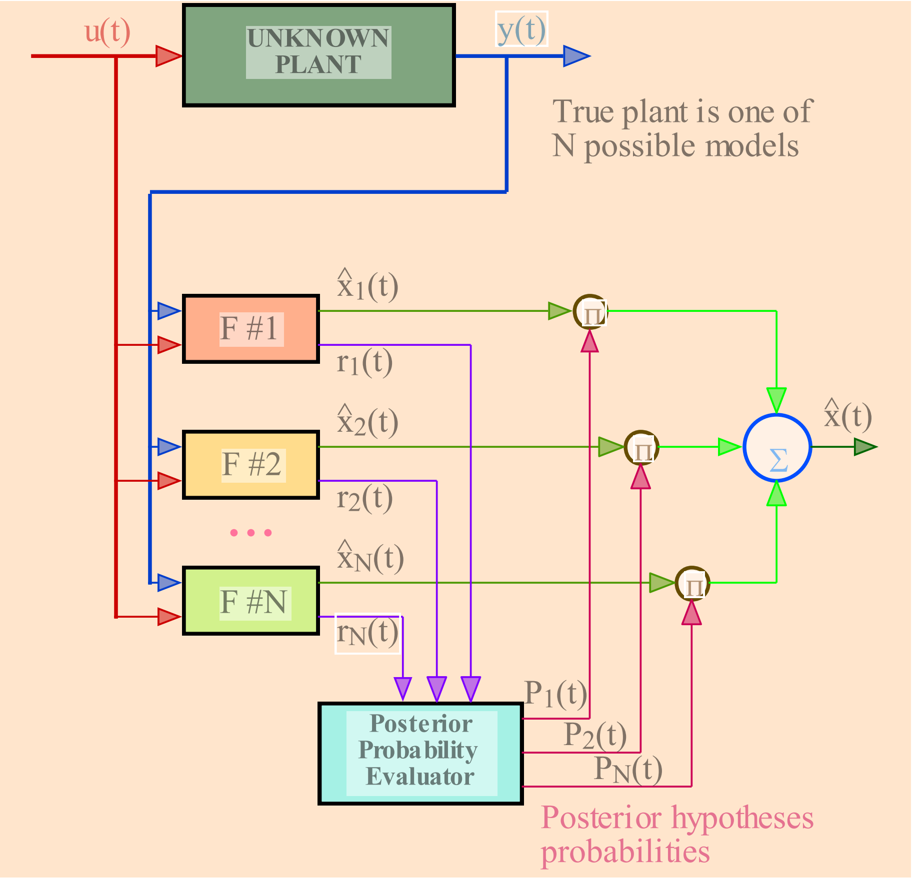
- Steel production
- Desired steel strength requires a certain % of carbon in iron
- Iron ore is mixed with carbon, calcium etc in pressurized vessel
- Superheated oxygen melts mixture and burns carbon
- Mass spectrometer measurements are used to estimate % carbon as a function of time





Combined Plant-Identification and State-Estimation

- Example of simultaneous system identification and state estimation
- True plant is one (or close to one) of N possible models
- A parallel bank of N filters is constructed, each corresponding to a specific model
- It is possible to evaluate the posterior probabilities of each model being the true plant
- Global state estimate is generated by probabilistic weighting the state estimates of each model



Concluding Remarks

- We must understand stochastic sequences and stochastic processes
- We must study the response of dynamic systems to uncertain initial states, modeled as random vectors
- We must study the response of dynamic systems to both deterministic time-functions and stochastic processes
- We must specify precisely the optimization philosophy of generating “best” estimates
- Real-time computational requirements dictate whether we use a truly optimal estimation algorithm or resort to a suboptimal one
  - solving in real-time differential or difference equations is feasible
  - solving in real-time partial (or integro-partial) differential equations is NOT feasible

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