REF. NO. KF #7

1
STOCHASTIC ESTIMATION

Numerical example: Effect of changing covariance matrix of measurement noise upon Kalman filter

2
MOTIVATION

- To illustrate
  - dependence of variance of state estimation errors
  - filter gains

upon different values of measurement covariance matrix $\underline{\Theta}$.

- A numerical example will be employed.

3
PROBLEM DEFINITION

- STATE DYNAMICS

$$\left.\begin{aligned} x_1(t+1) &= x_2(t)+2x_4(t) \\ x_2(t+1) &= -4x_1(t)+x_3(t) \\ x_3(t+1) &= x_4(t) \\ x_4(t+1) &= -x_1(t)+x_3(t)+\varepsilon_4(t) \end{aligned}\right\} \rightarrow \underline{A} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ -4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \qquad (1)$$

- MEASUREMENT EQUATION

$$z(t) = x_1(t)+\theta(t) \quad \rightarrow \quad \underset{\sim}{\underline{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad (2)$$

4
PROBABILISTIC INFORMATION

- INITIAL STATE COVARIANCE MATRIX

$\underline{\Sigma}(0/0) = \underline{\Sigma}(0)$

$$\underline{\Sigma}(0) = \begin{bmatrix} 25 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix} \qquad (3)$$

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• PLANT NOISE COVARIANCE MATRIX $\underline{\Xi}\delta_{t\tau}$

$$\text{cov}\left[\underline{\xi}(t);\underline{\xi}(\tau)\right] = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{\Xi}}\delta_{t\tau} \qquad (4)$$

6
• MEASUREMENT NOISE COVARIANCE MATRIX $\underline{\Theta}\delta_{t\tau}$

$$\text{cov}\left[\underline{\theta}(t);\underline{\theta}(\tau)\right] = \underline{\Theta}\delta_{t\tau} \qquad (5)$$

$$\underline{\Theta} = \Theta = \text{scalar} \qquad (6)$$

$$\underline{\Theta} = \left.\begin{matrix} 0.001 \\ 0.25 \\ 1.0 \\ 10.0 \\ 100.0 \end{matrix}\right\} \text{These were values used} \qquad (7)$$

7
STRUCTURE OF UPDATED COVARIANCE MATRIX $\underline{\Sigma}(t/t)$

$$\underline{\Sigma}(t/t) = \begin{bmatrix} \Sigma_{11}(t/t) & 0 & \Sigma_{13}(t/t) & 0 \\ 0 & \Sigma_{22}(t/t) & 0 & \Sigma_{24}(t/t) \\ \Sigma_{13}(t/t) & 0 & \Sigma_{33}(t/t) & 0 \\ 0 & \Sigma_{24}(t/t) & 0 & \Sigma_{44}(t/t) \end{bmatrix} \qquad (8)$$

8
STRUCTURE OF FILTER GAIN MATRIX

$$\underline{H}(t) = \underline{\Sigma}(t/t)\,\underline{C}'\,\underline{\Theta}^{-1} = \begin{bmatrix} h_1(t) \\ 0 \\ h_3(t) \\ 0 \end{bmatrix} \qquad (9)$$

9

STATE ESTIMATE STRUCTURE

• Predict Equation

$$\left.\begin{aligned}
\hat{x}_1(t+1/t) &= \hat{x}_2(t/t) + 2\hat{x}_4(t/t)\\
\hat{x}_2(t+1/t) &= -4\hat{x}_1(t/t) + \hat{x}_3(t/t)\\
\hat{x}_3(t+1/t) &= \hat{x}_4(t/t)\\
\hat{x}_4(t+1/t) &= -\hat{x}_1(t/t) + \hat{x}_3(t/t)
\end{aligned}\right\} \qquad (10)$$

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• Update Equation

$$\left.\begin{aligned}
\hat{x}_1(t+1/t+1) &= \hat{x}_1(t+1/t) + h_1(t+1)\left[z(t+1) - \hat{x}_1(t+1/t)\right]\\
\hat{x}_2(t+1/t+1) &= \hat{x}_2(t+1/t)\\
\hat{x}_3(t+1/t+1) &= \hat{x}_3(t+1/t) + h_3(t+1)\left[z(t+1) - \hat{x}_1(t+1/t)\right]\\
\hat{x}_4(t+1/t+1) &= \hat{x}_4(t+1/t)
\end{aligned}\right\} \qquad (11)$$

KF gains

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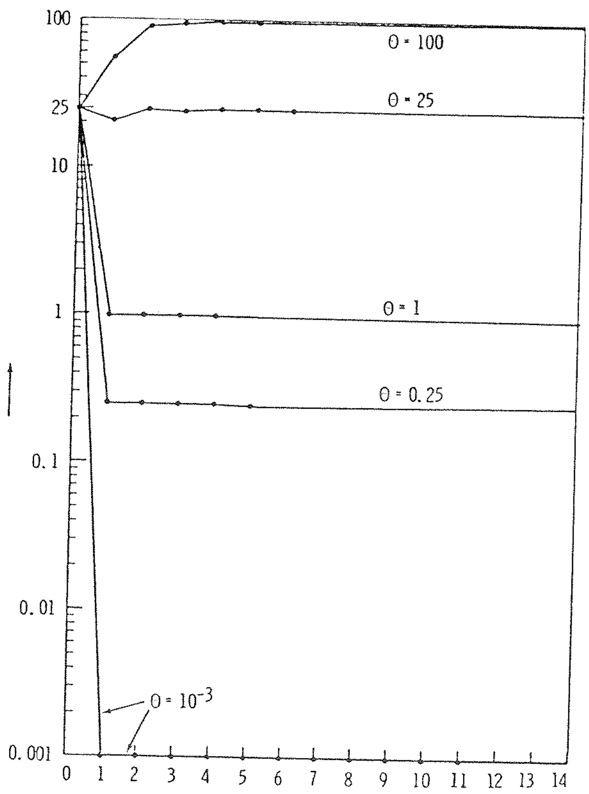

VARIATION OF $\Sigma_{11}(t/t)$ FOR DIFFERENT VALUES OF θ

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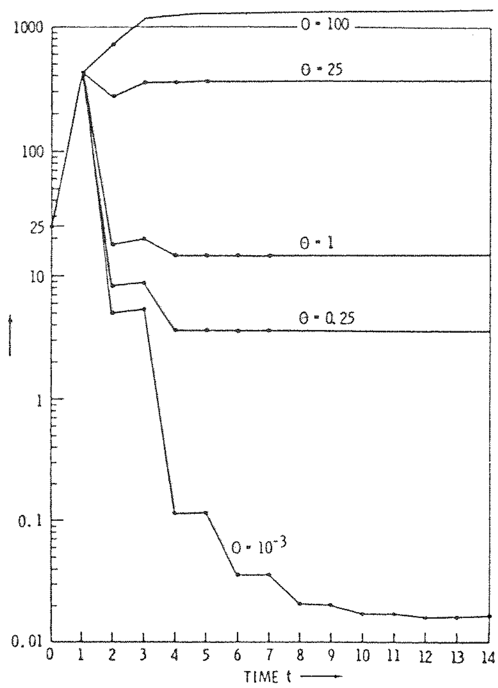
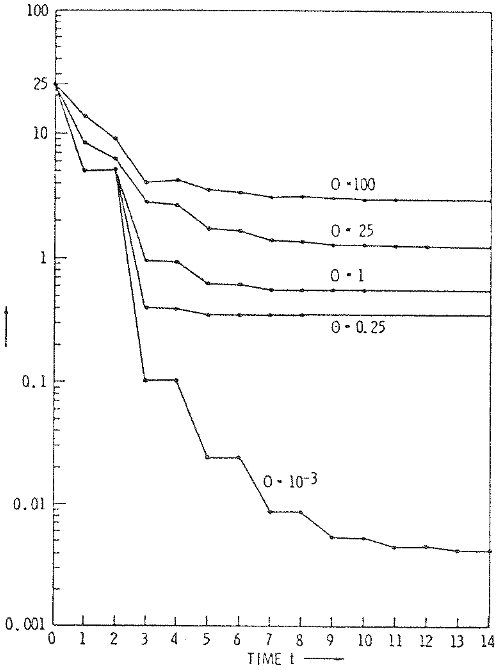


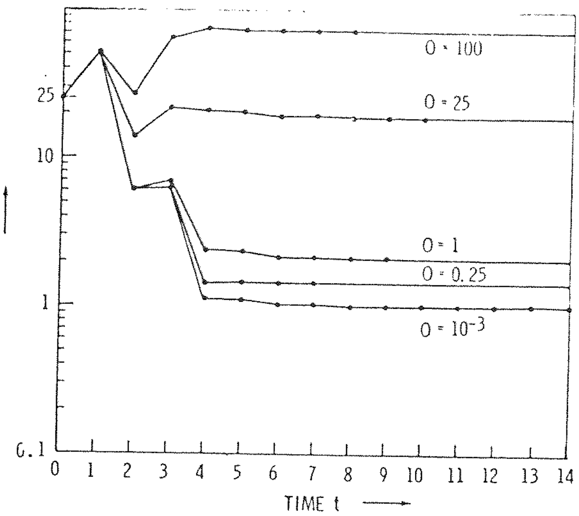
VARIATION OF $\Sigma_{22}(t/t)$ FOR DIFFERENT VALUES OF θ

13



VARIATION OF $\Sigma_{33}(t/t)$ FOR DIFFERENT VALUES OF θ

14



VARIATION OF $\Sigma_4(t/t)$ FOR DIFFERENT VALUES OF 0

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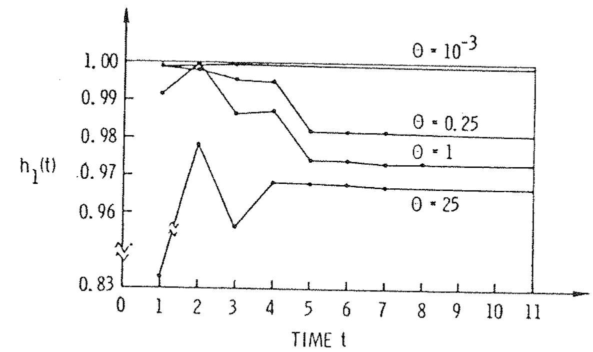
VARIATION OF FILTER GAIN $h_1(t)$



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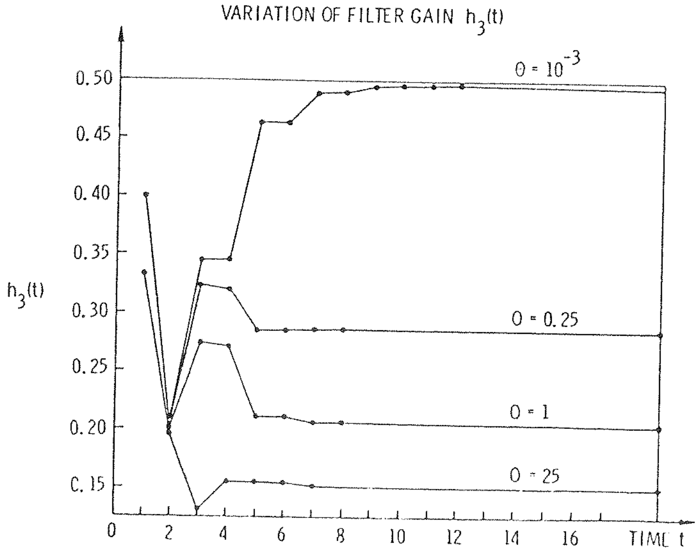
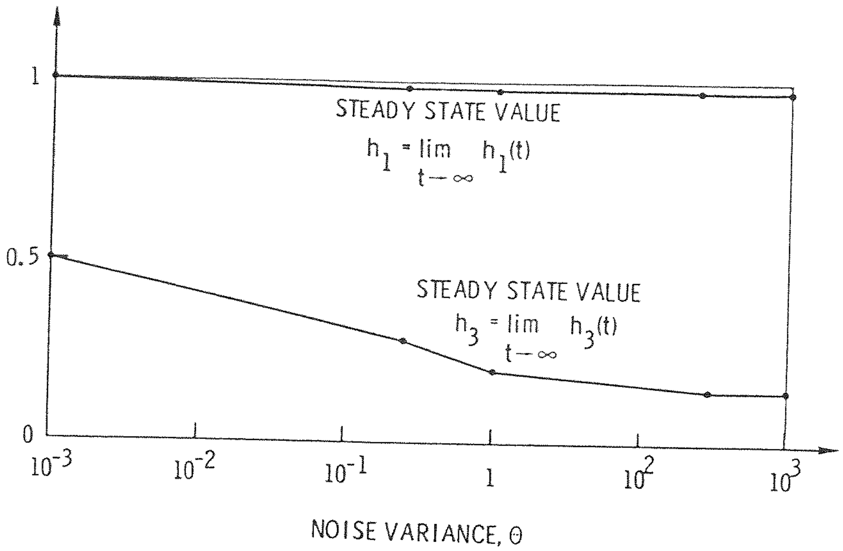
VARIATION OF FILTER GAIN $h_3(t)$





KF gains are reasonably "robust"
to "small errors" in noise ~~variance~~ intensity (H)