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Online Model Identification for Set-valued State Estimators With Discrete-Time Measurements

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Introduction/ Motivation

The Kalman Filter framework is a powerful tool for state estimation in linear systems;

The main assumption is that the system model is known with sufficient accuracy;

- ***Lack of robustness to parameter variations*** **(performance loss);**

The goal of this work is to exemplify the application of a robust filter, based on the Kalman Filter;

- ***State estimation and system identification.***

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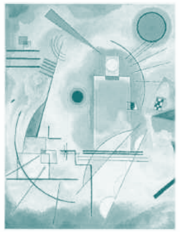
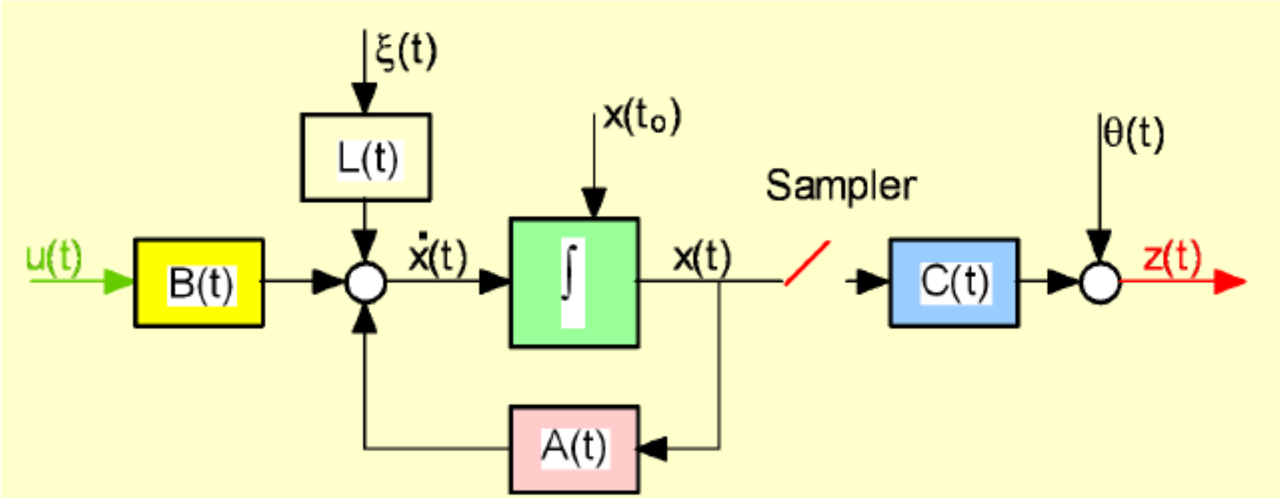


Robust State Estimation



$$A(t) = \hat{A}(t) + \Delta_{\alpha}(t)$$

$$B(t) = \hat{B}(t) + \Delta_{\beta}(t)$$

$$C(t) = \hat{C}(t) + \Delta_{\gamma}(t)$$

Parameter variations have to be taken explicitly into account.

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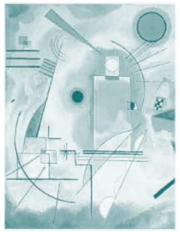

Robust State Estimation

Robust Estimation Methods:
- Guaranteed Cost Estimators;
- Set-Valued Estimators;
- Adaptive Estimators (eg. MMAE);

Approaches differ in the way that parameter uncertainty is described.

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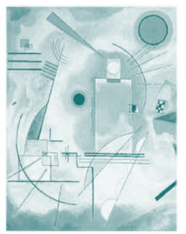

Set-Valued Estimation (Petersen & Savkin, 1995)

Main characteristics:

-Estimates the set of possible states for a given instant;

-Deterministic interpretation of system uncertainty;

-Allows for non-linear, time-varying uncertainties;

-Allows for continuous / discrete sensors;

-Allows for missing data;

- Model validation can be performed as a dual problem.

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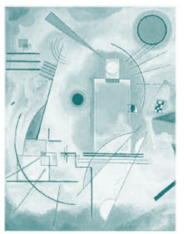
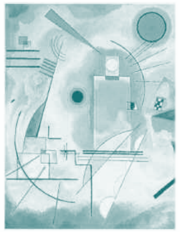


Set-Valued Estimation

$$
\begin{aligned}
\dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u(t);\\
z_c(t) &= K_c(t)x(t) + G_c(t)u(t).\\
z_d(t_j) &= K_d(t_j)x(t_j) + G_d(t_j)u(t_j)\ \forall j = 1, 2, \ldots, N_d;\\
y_c(t) &= C_c(t)x(t) + v_c(t);\\
y_d(t_j) &= C_d(t_j)x(t_j) + v_d(t_j)\ \forall j = 1, 2, \ldots, N_d
\end{aligned}
$$

Integral Quadratic Constraint:

$$
\begin{aligned}
&(x(0) - x_0)' N (x(0) - x_0) + \int_0^s \left(w(t)' Q(t) w(t) + \boldsymbol{v_c(t)' R_c(t) v_c(t)} \right) \boldsymbol{dt}\\
&\quad + \sum_{t_j \le s} v_d(t_j)' R_d(t_j) v_d(t_j) \ \le d + \int_0^s \|z_c(t)\|^2 dt + \sum_{t_j \le s} \|z_d(t_j)\|^2 .
\end{aligned}
$$



Set-Valued Estimation

Ricatti *jump* equations:

$$
\begin{aligned}
\dot{P}(t) &= A(t)P(t) + P(t)A(t)' + B_1(t)Q(t)^{-1}B_1(t)' \\
&\quad + P(t)[K_c(t)'K_c(t) - C_c(t)'R_c(t)C_c(t)]P(t) \\
&\quad \textit{for } t \neq t_j;
\end{aligned}
$$

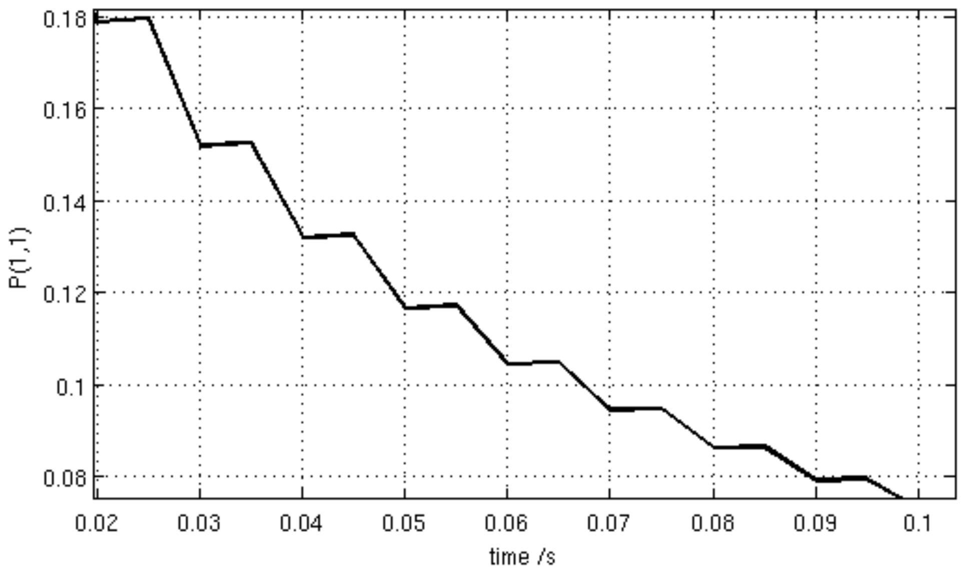
$$
\begin{aligned}
P(t_j) &= \left[\; P(t_j^-)^{-1} + C_d(t_j)'R_d(t_j)C_d(t_j) - K_d(t_j)'K_d(t_j) \;\right]^{-1} \\
&\quad \textit{for } j = 1, 2, \ldots, N_d.
\end{aligned}
$$

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \left[\; A(t) + P(t)[K_c(t)'K_c(t) - C_c(t)'R_c(t)C_c(t)] \;\right]\hat{x}(t) \\
&\quad + P(t)C_c(t)'R_c(t)y_c^0(t) \\
&\quad + [P(t)K_c(t)'G_c(t) + B_2(t)]u^0(t) \;\; \textit{for } t \neq t_j;
\end{aligned}
$$

$$
\begin{aligned}
\hat{x}(t_j) &= \hat{x}(t_j^-) + P(t_j^-)\left[\; K_d(t_j)'K_d(t_j) - C_d(t_j)'R_d(t_j)C_d(t_j) \;\right]\hat{x}(t_j^-) \\
&\quad + P(t_j^-)C_d(t_j)'R_d(t_j)y_d^0(t_j) + P(t_j^-)K_d(t_j)'G_d(t_j)u^0(t_j) \\
&\quad \textit{for } j = 1, 2, \ldots, N_d.
\end{aligned}
$$

Set-Valued Estimation

Solution isn't smooth due to discrete-time measurements.



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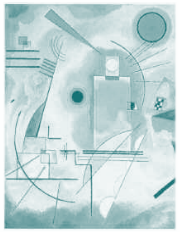

Set-Valued Estimation

Set estimation:

$$
\begin{aligned}
&X_s[x_0, u^0(\cdot)|_0^s, y_c^0(\cdot)|_0^s, y_d^0(\cdot)|_0^s, d] \\
&= \left\{ \begin{array}{ll} x_s \in \mathbf{R}^n : & (x_s - \hat{x}(s))' P(s)^{-1} (x_s - \hat{x}(s)) \\ & \leq d + \rho_s[u^0(\cdot), y_c^0(\cdot), y_d^0(\cdot)] \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
&\rho_s[u^0(\cdot), y_c^0(\cdot), y_d^0(\cdot)] \\
&\triangleq \int_0^s \left[\begin{array}{l} \|(K_c(t)\hat{x}(t) + G_c(t)u^0(t))\|^2 \\ -(C_c(t)\hat{x}(t) - y_c^0(t))' R_c(t) (C_c(t)\hat{x}(t) - y_c^0(t)) \end{array} \right] dt \\
&\quad + \sum_{t_j \leq s} \left[\begin{array}{l} \|(K_d(t_j)\hat{x}(t_j) + G_d(t_j)u^0(t_j))\|^2 \\ -(C_d(t_j)\hat{x}(t_j) - y_d^0(t_j))' R_d(t_j) (C_d(t_j)\hat{x}(t_j) - y_d^0(t_j)) \end{array} \right]
\end{aligned}
$$

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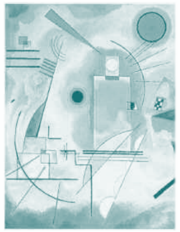
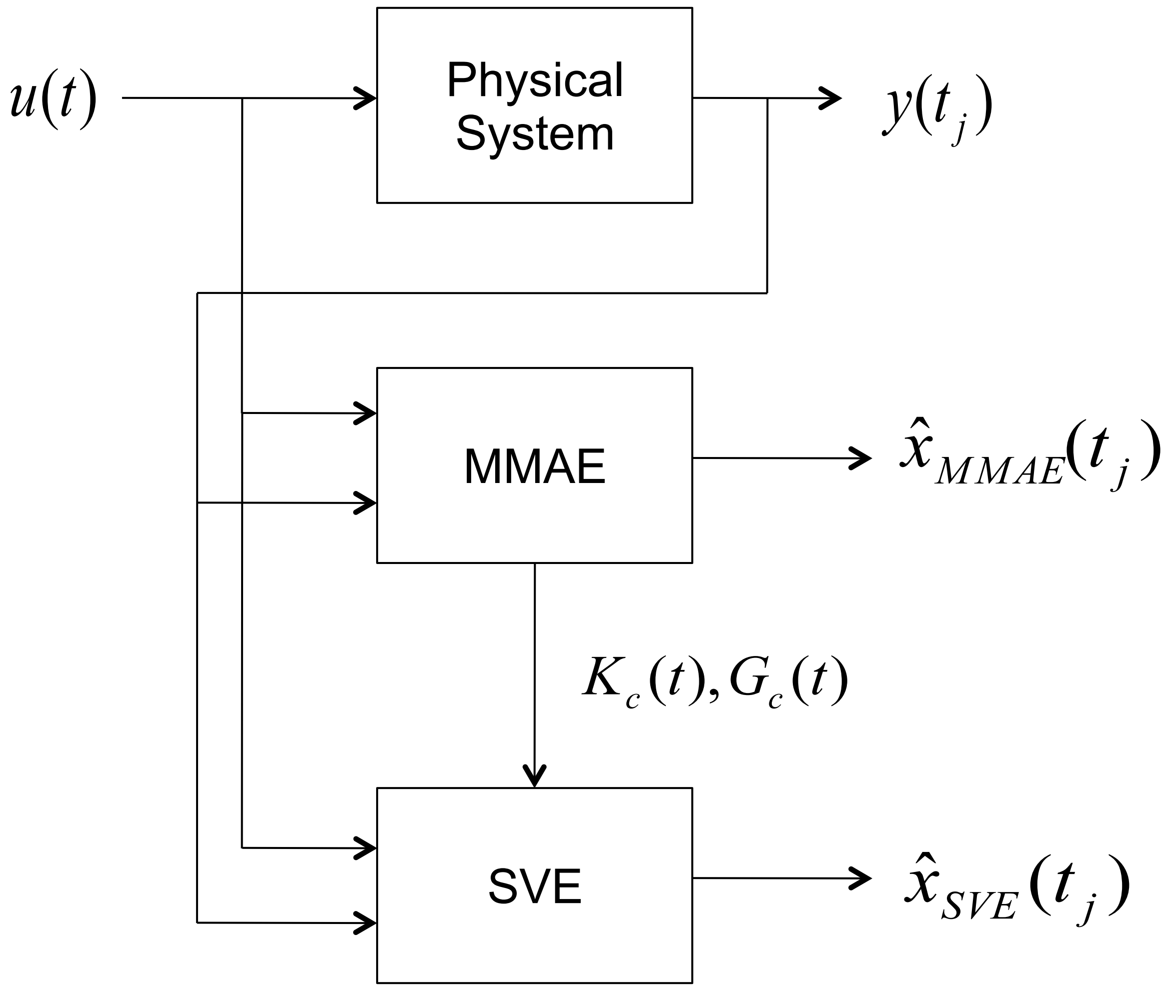


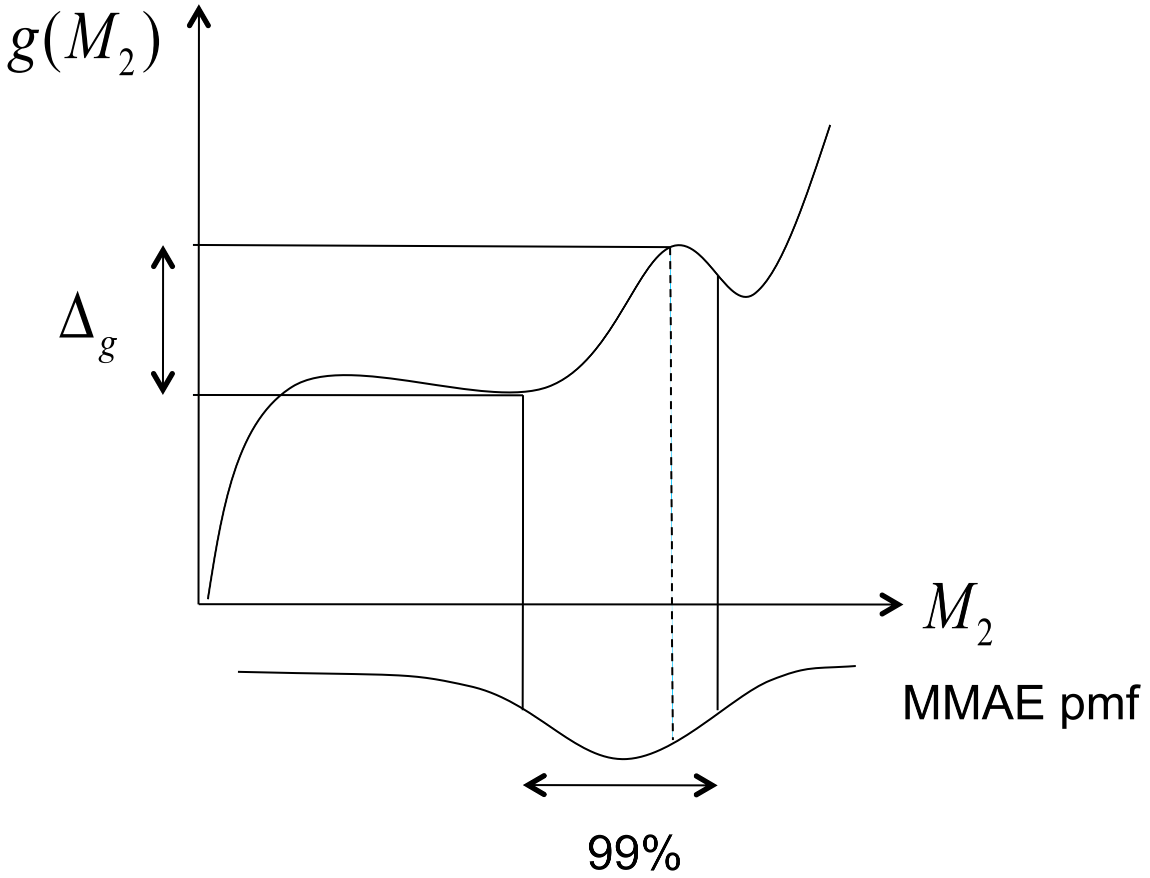
System Identification

- **This method allows for dynamic re-evaluation of system uncertainty.**
- **However, this only checks if a given model is feasible or not;**
- **No explicit methodology for system identification is given;**
- **MMAE is used to compensate;**

Implementation



Implementation



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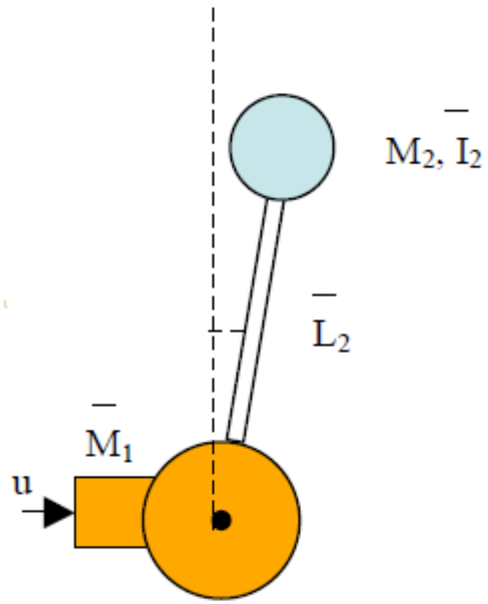
Implementation: HTS



$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$z_c(t) = \begin{bmatrix} k_\alpha(t) & 0 \end{bmatrix} x(t) + k_\beta(t) u(t)$$

$$y_d(t_j) = x(t_j) + v_d(t_j)$$

$$w(t) = \Delta z_c(t)$$

$$\|\Delta\| \leq 1 \quad R_d(t_j) = I \quad Q(t) = I$$

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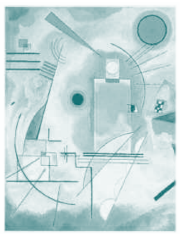
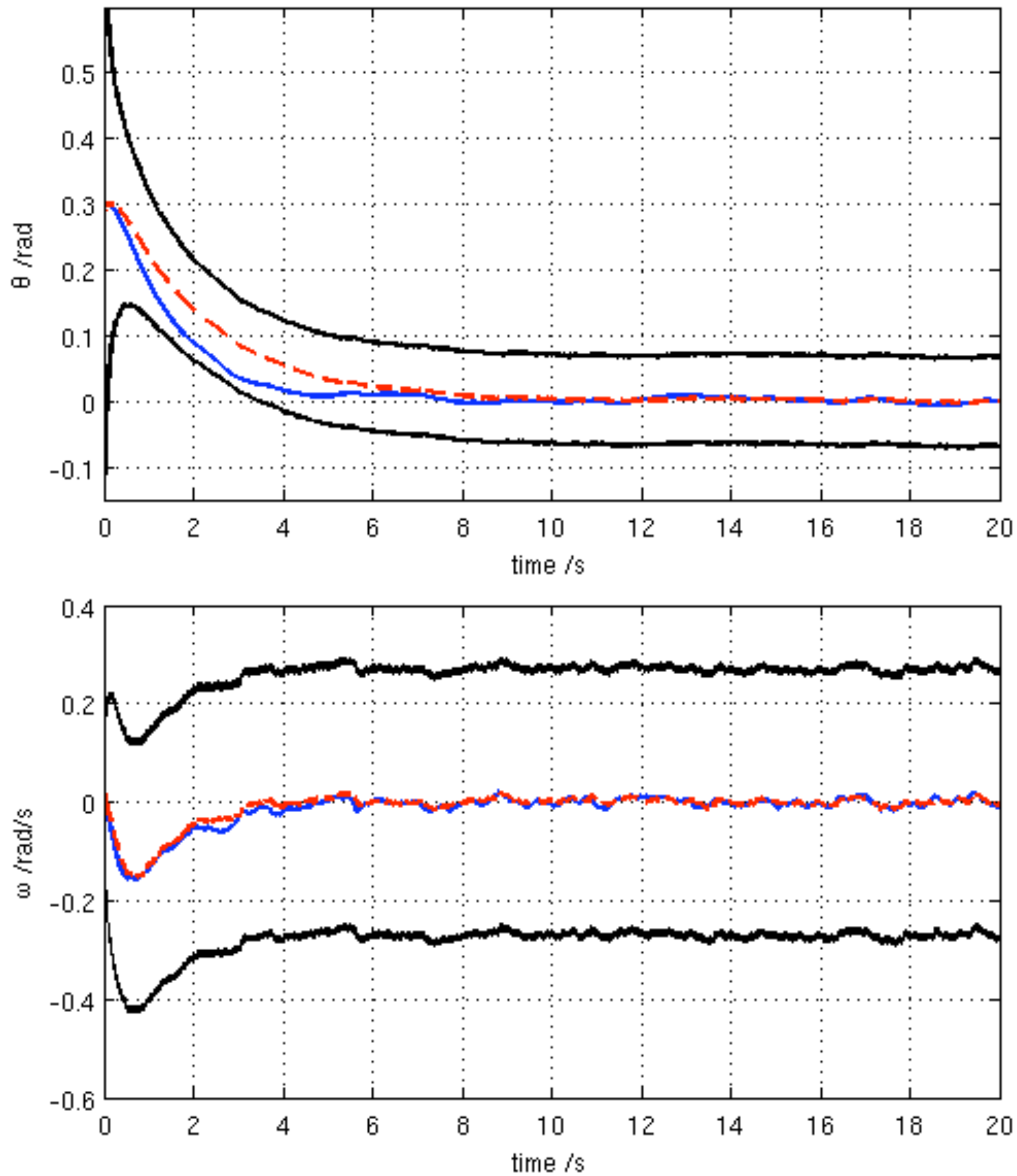


Results



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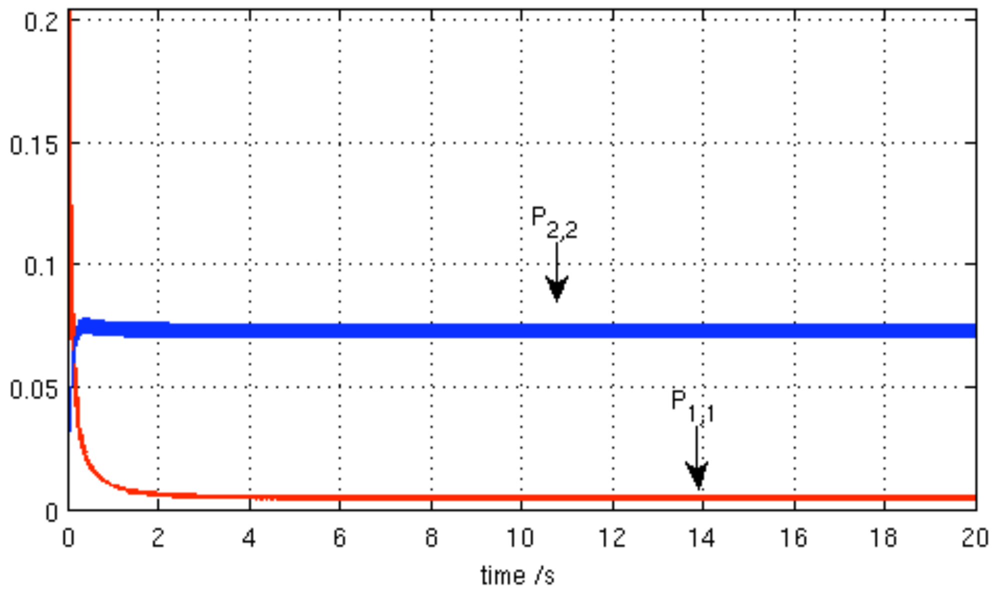
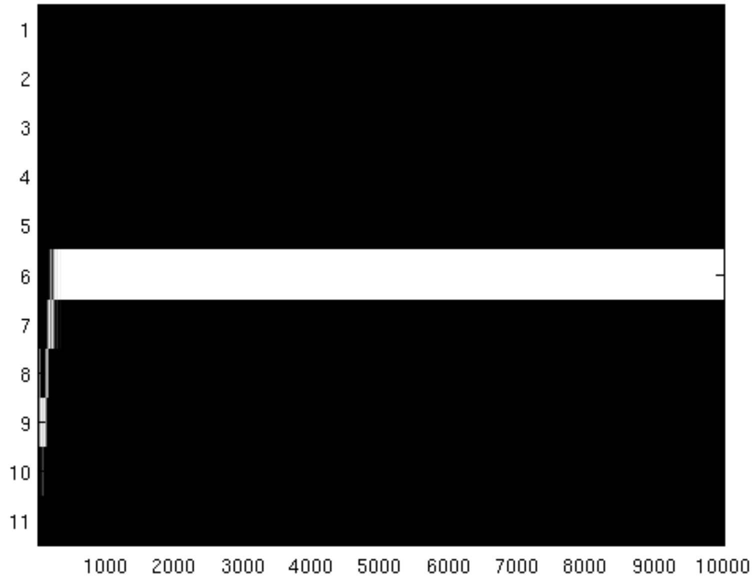
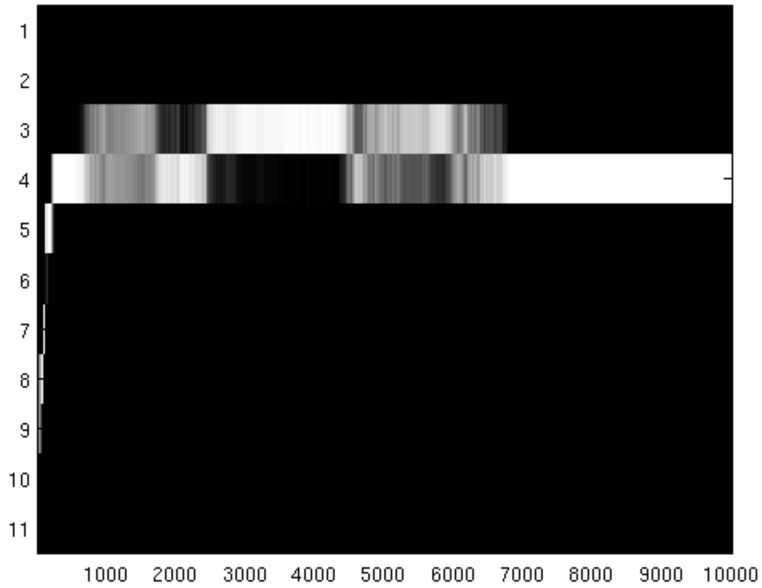
Results



Results

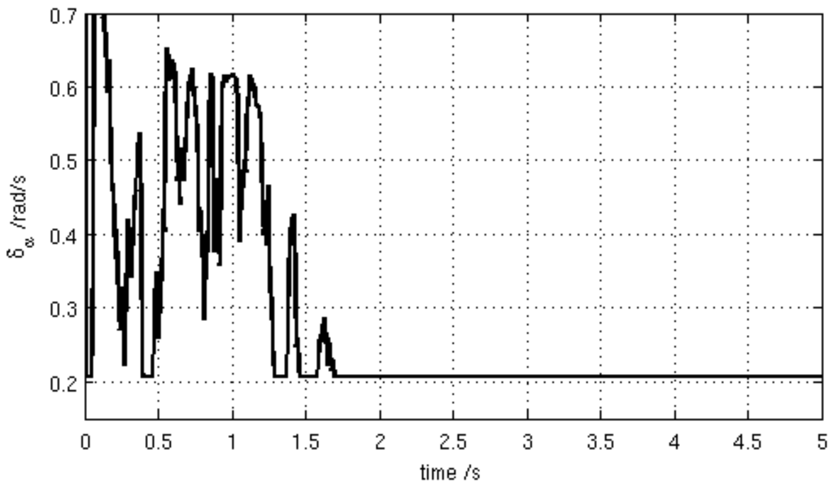


M2 = 75kg

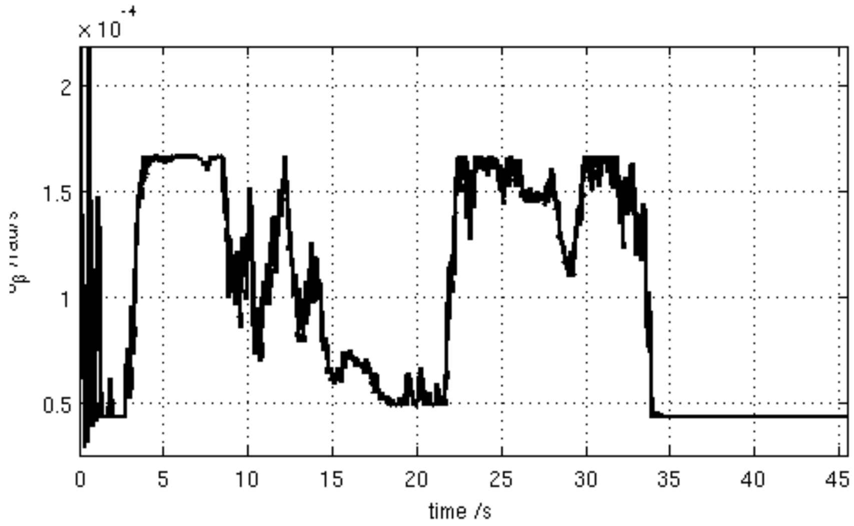


M2 = 65kg

Results


0.7
0.6
0.5
0.4
0.3
0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time /s
δα /rad/s

α


× 10-4
2
1.5
1
0.5
0
5
10
15
20
25
30
35
40
45
time /s

β

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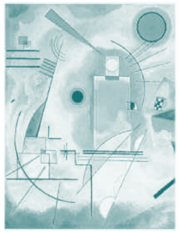
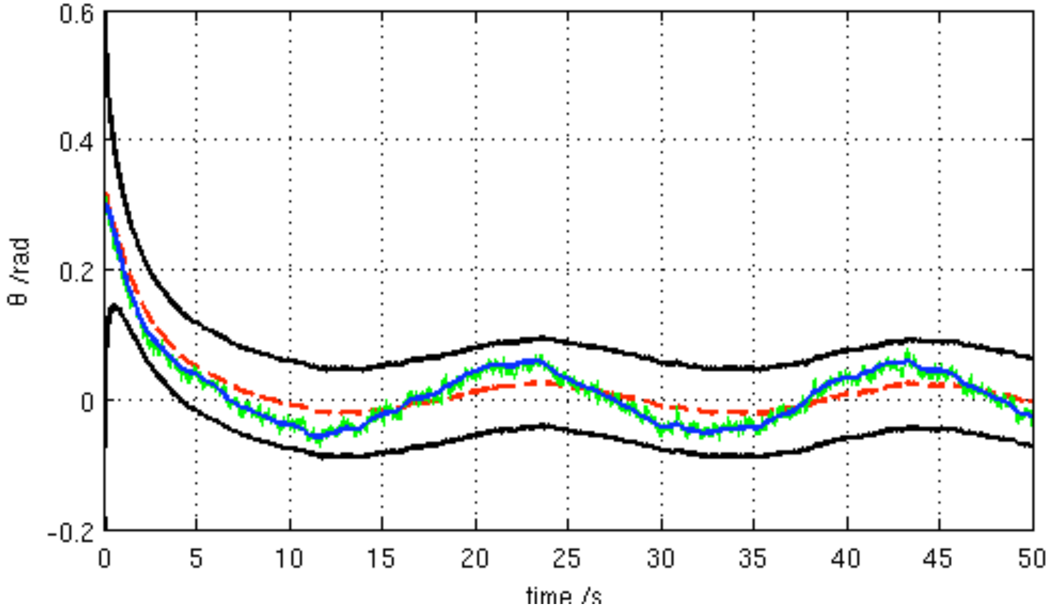
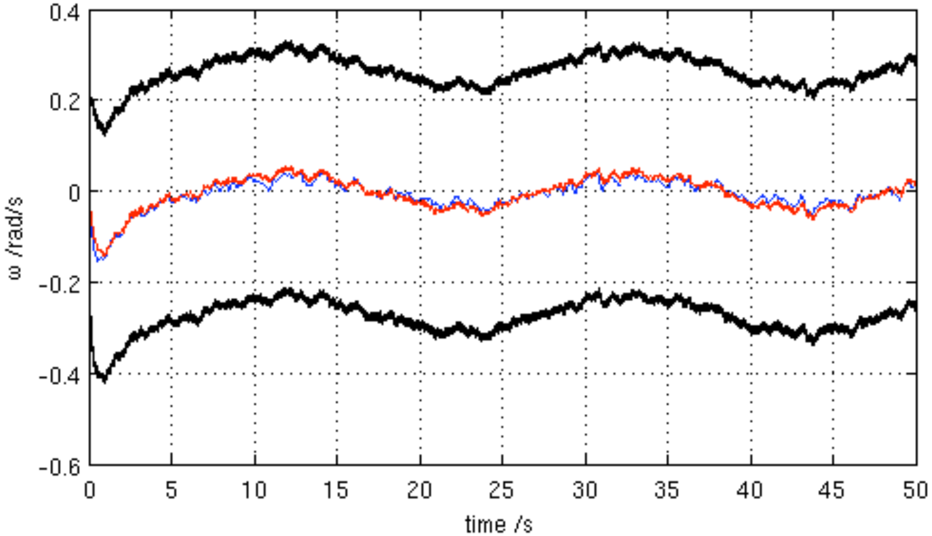


Results





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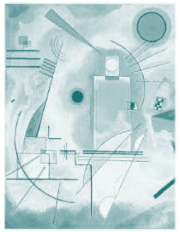

Conclusions

- **Main advantage of set-valued estimators: deals with a large class of uncertainties;**
- **Not straightforward to implement;**
- **Performance is dependant on the quality of the norm bounds on the parameters;**
- **If some of these bounds are configured simply by user design, the results are overly conservative;**
- **It is possible to perform system identification to reduce these norm bounds on-line;**
- **It may be feasible to implement some form of particle-filter based parameter search.**