Nonlinear Wind Estimator Based on Lyapunov Techniques

Pedro Serra[†]

[†]ISR/DSOR

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Outline

Introduction

Motivation

- Difficulty to accurately measure the vehicle's position regarding the local environment
  - GPS provides positioning in the (ECEF) without considering local topography
  - measurement rate is not sufficient for some applications
  - quality of the height's measure is poor
  - signals are subjected to shortages some environments

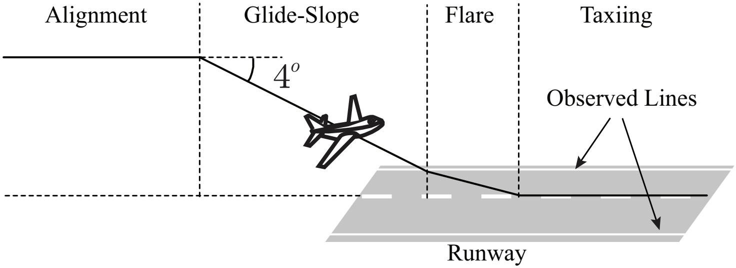
Introduction

Motivation

- Using cameras as primary sensors for relative position allows to cast the problem into an Image-Based Visual Servo Control
  - possibility to perform autonomous tasks in low-structured environments with no external assistance
  - A vision system can be cheap, light and adaptable
  - can be used to provide robust relative pose information

Introduction

Problem



- Tracking parallel linear visual features though IVBS control
- Estimate the wind velocity in the orthogonal direction to the linear visual features

Modeling

Aircraft Dynamics

$$v = v_a + v_w$$

$$\begin{aligned} \dot{\xi} &= R(v_a + v_w) \\ m\dot{v} &= -\Omega_\times mv + F_g + F_{aero} + F_{engine} \\ \dot{R} &= R\Omega_\times \\ I\dot{\Omega} &= -\Omega_\times I\Omega + \Gamma_{aero} \end{aligned}$$

$$F_{aero} = QS \begin{bmatrix} C_X(\alpha, \beta) & C_{Y_\beta}\beta & C_{Z_\alpha}(\alpha - \alpha_0) \end{bmatrix}^T$$

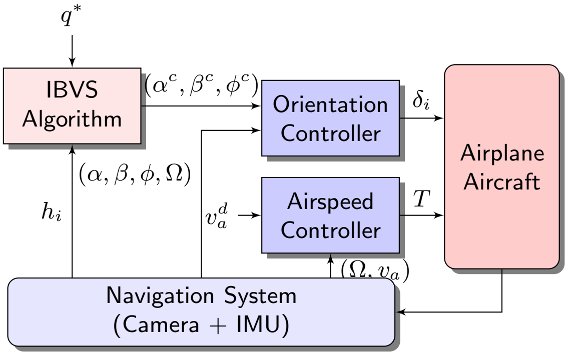
Modeling

Guidance, Control and Navigation structure

- regulating the norm of the airspeed $V_a = \|v_a\|$ on a desired velocity V_a^d;
- regulating the sideslip angle β to zero
- stabilizing the orientation dynamics through a fast inner-loop controller such that assignments in (ϕ, α, β) are correctly stabilized
- stabilizing the translational dynamics by considering $\beta = 0$ and V_a constant using (ϕ, α) as control inputs.

Modeling

Guidance, Control and Navigation structure



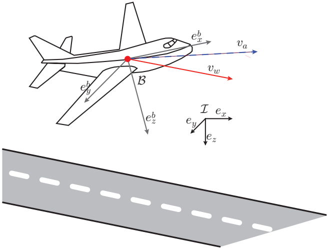
Modeling

Guidance Dynamics

$$\dot{\xi} = R(v_a + v_w) \quad (1)$$

$$\dot{v}_w = -\Omega_\times v_w \quad (2)$$

$$\dot{v}_a = -\Omega_\times v_a + \pi_{v_a} u_a(\alpha, \phi) \quad (3)$$



$$\pi_{v_a} = I_d - \frac{v_a v_a^T}{V_a^2}$$

$$u_a(\alpha, \phi) = mg\cos\theta\sin\phi e_y + QSC_{Z_\alpha}(\alpha - \alpha_0)e_\alpha$$

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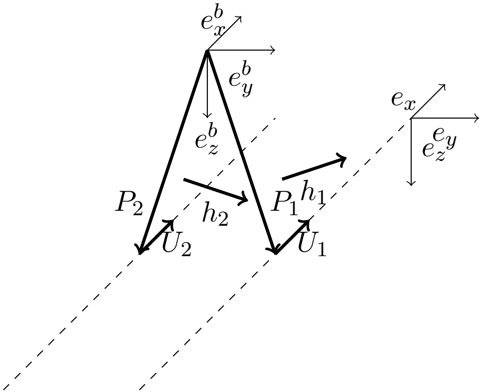
Geometrical Properties



- Linear features representation

$$h_i = \frac{P_i \times U_i}{\|P_i \times U_i\|}$$

- Centroid

$$q = h_1 + h_2$$

- Desired centroid

$$q* = R^T b^*$$

- Error vector

$$\delta = q - q^*$$

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Error Dynamics

- Error dynamics

$$\dot{\delta} = -\Omega_\times \delta - Q[(v_a + v_w) \times U]$$

- Let $\hat{v}_w$ be an estimative of $v_w \times U$

$$\tilde{v}_w = v_w \times U - \hat{v}_w$$

- Then

$$\dot{\delta} = -\Omega_\times \delta + Q[\mathrm{sk}(U) v_a - \hat{v}_w - \tilde{v}_w)]$$

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Wind Estimator

- Need to ensure $\|\hat{v}_w\| < \varepsilon V_a$, then

$$\hat{v}_w = \varepsilon' V_a \frac{y}{\sqrt{1 + \|y\|^2}}$$

- Choosing the dynamics for y as an external pertubation constant in the inertial frame

$$\dot{y} = -\Omega_\times y + \pi_U u_w, \qquad y(0) = 0$$

where u_w acts as the innovation of the wind estimator.

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Wind Estimator

- After some tedious computations

$$\dot{\hat{v}}_w = -\Omega_\times v_w + P_{\hat{v}_w} \pi_U u_w$$

$$P_{\hat{v}_w} = \varepsilon' V_a \sqrt{1 - \frac{\|\hat{v}_w\|^2}{\varepsilon'^2 V_a^2}} \left(I - \frac{\hat{v}_w \hat{v_w}^T}{\varepsilon'^2 V_a^2} \right), \quad \varepsilon' \in \left(\frac{1+\varepsilon}{2}, 1 \right).$$

- The estimation remains in the plane orthogonal to U.

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Control Design

- Let v_a^d be the desired airspeed

$$v_a^d = -\text{sk}(U)(k_1\delta - \hat{v}_w) + \sqrt{V_a^2 - \|k_1\delta - \hat{v}_w\|^2}U$$

- Define a second error term

$$\delta_2 = v_a - v_a^d$$

- and let S_1 and S_2 be two storage functions

$$S_1 = \|\delta\|^2 + \frac{2}{k_1}\tilde{v}_w^T\delta + \frac{4}{k_1^2}\|\tilde{v}_w\|^2 \qquad S_2 = \frac{\|\delta_2\|^2}{2}$$

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Control Design

Theorem

Specify the inovation term of the wind estimate and the control input as

$$u_w = k_2 \delta \qquad u_a = \frac{2k_3 V_a}{\|v_a + v_a^d\|^2} \pi_{v_a} v_a^d$$

then there exists positive gains (k_1, k_2, k_3, K) *such that the function*

$$L = S_1 + K S_2$$

is a Lyapunov function for the guidance dunamics that guarantees that the error signals $(\delta, \delta_2, \tilde{v}_w)$ *converge exponentially to zero.*

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Control Design - proof

Proof.

Computing the time derivative of S_1 and S_2 and applying the control input an the innovation term one can show that

$$\begin{aligned}\dot{L} \leq & -\left(\delta + \frac{\tilde{v}_w}{k_1}\right)^T M_1(k_i, K, Q, P_{\hat{v}_w})\left(\delta + \frac{\tilde{v}_w}{k_1}\right) \\ & -g_1(k_2, k_1, K)\delta^T P_{\hat{v}_w}\delta \\ & -k_3 K\|\delta_2\|^2 - K\delta_2^T M_2(U, Q, k_i, v_a^d)\delta_2\end{aligned}$$

where it is possible to choose positive gains (k_1, k_2, k_3, K) such that $M_1(k_i, K, Q, P_{\hat{v}_w})$ and $M_2(U, Q, k_i, v_a^d)$ are positive defenite matrices and $g_1(k_2, k_1, K)$ is a positive scalar. □

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Control Design - proof

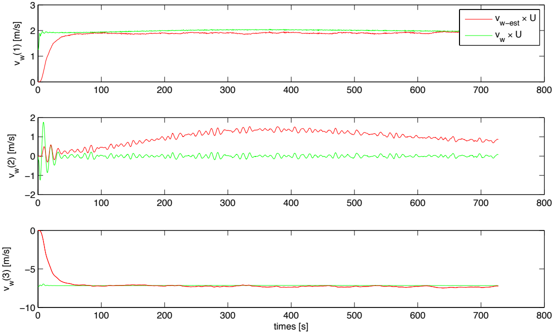
Proof - cont.

- Lyapunov direct method ensures that $(P_{\hat{v}_w}\delta, \delta + k_1^{-1}\tilde{v}_w, \delta_2)$ converge towords zero.
- Note that is $P_{\hat{v}_w}$ positive as soon as $\tilde{v}_w < \varepsilon' V_a$
- and due to the restrictions imposed for the gain k_1, $\delta + k_1^{-1}\tilde{v}_w = 0$ implies that $\|v_w\| < \varepsilon' V_a$
- Thus $P_{\hat{v}_w}$ is a positive definite matrix at the equilibrium, implying that $(\delta, \tilde{v}_w, \delta_2)$ converge to zero exponentially.

□

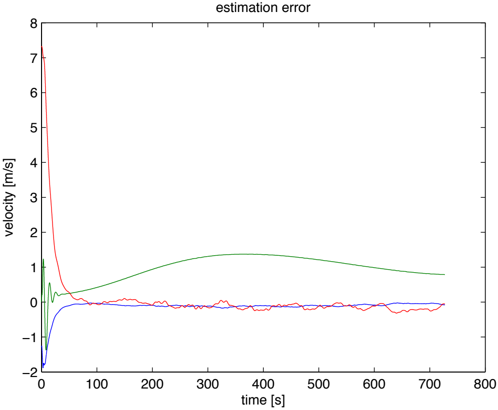
Results

Wind Estimator



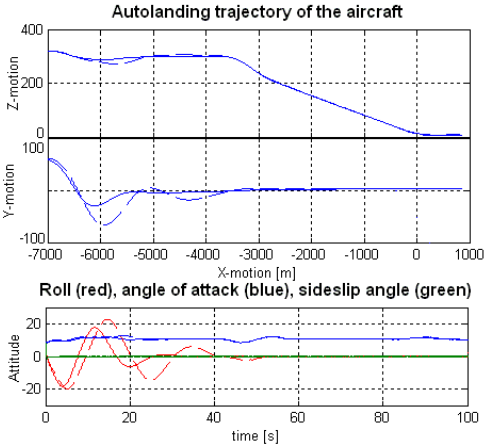
Results

Estimation Error



Results

Landing Maneuver



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