Fault Detection and Isolation of an Aircraft Using Set-Valued Observers ⋆

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Abstract: This paper describes an application of a new fault detection and isolation (FDI) technique based on set-valued observers (SVOs) to a linear parameter varying (LPV) longitudinal aircraft dynamic model. The FDI strategy adopted herein computes and uses the set-valued estimates of the SVOs to *eliminate* models of the plant that are not compatible with the set of observations provided by the aircraft sensor suite and actuation data. The design of the SVOs takes into account model uncertainty and disturbances, thus avoiding false alarms due to such perturbations. The behavior of the proposed solution is assessed in simulation, by deliberately generating hard and soft sensor/actuator faults. The results show that the faults take, in general, only a few iterations to be detected and isolated, therefore paving the way for the use of the proposed methodology in practical applications.

Keywords: Fault Detection and Isolation, Uncertain Linear Systems, Linear Time-Varying Systems

1. INTRODUCTION

The field of Fault Detection and Isolation (FDI) has been one of the focus of attention of the control systems identification communities since the early 70's Willsky (1976). Indeed, several types of approaches can be found in the literature and have been implemented and tested during the last decade – see, for instance, Blanke et al. (1997, 2001); Isermann (1997); Patton and Chen (1997); Frank and Ding (1997); Esteban (2004); Collins and Tinglun (2001); Longhi and Moteriù (2009); Mattone and De Luca (2006). Recently, a novel strategy for *model invalidation* has been introduced in Rosa et al. (2009), which can be applied to FDI of linear time-varying (LTV) plants, and that possesses several important properties in *safety critical* scenarios, as discussed in the sequel. Model invalidation or model falsification is used to *eliminate* models of the plant that are not compatible with the data acquired from the sensors. As shown in this paper, this method can be used for FDI. One of the main concerns common to all FDI systems is that model uncertainty (such as unmodeled dynamics) and disturbances should never be interpreted as faults, and indeed the approach in Rosa et al. (2009) is able to account for this type of uncertainty.

Among the different Fault Detection (FD) and FDI approaches available in the literature (see, for instance, Esteban (2004) for an in-depth presentation of such methods), active and deterministic model-based fault detection strategies are of particular interest, due to their ability to explicitly handle robustness problems and of detecting faults within a small time-window. Classical approaches to this type of fault detection strategies are usually composed of two parts: a filter that generates residuals, which should be *large* under faulty behaviors; and a decision threshold, which is used to decide whether a fault is present or not – see Patton and Chen (1997); Frank and Ding (1994); Esteban (2004); Massoumnia (1986); Willsky (1976); Besançon (2003); Bokor and Balas (2004); Meskin and Khorasani (2009); Wang et al. (2009); Narasimhan et al. (2008) and references therein.

The main idea in such approaches is, therefore, to design filters that are significantly more sensitive to faults than to disturbances and model uncertainty. This can be achieved, for instance, by using geometric considerations regarding the plant Massoumnia (1986); Longhi and Moteriù (2009); Bokor and Balas (2004), or by considering norm-optimization based methods Edelmayer et al. (1994); Frank and Ding (1997); Niemann and Stoustrup (2001); Marcos et al. (2005); Collins and Tinglun (2001). The later approach provides, in general, important robustness properties, as stressed in Edelmayer et al. (1994); Mangoubi et al. (1995); Patton and Chen (1997); Esteban (2004), by explicitly accounting for model uncertainty. The isolation of the fault can, in some cases, be done using a similar approach, *i.e.*, by designing filters for families of faults, and identifying the most likely fault as that associated to the filter with smaller residuals.

The FDI strategy adopted in this paper uses a different philosophy. Instead of identifying the most likely model of the nominal/faulty plant, we discard models that are not compatible with the observations. As shown in the sequel, this method guarantees that there will not be false alarms, as long as the model of the non-faulty plant remains valid. Moreover, we need not address the difficult problem of computing the decision threshold used to declare whether or not a fault has occurred. To this end, we use the set-valued

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observers (SVOs) – see Witsenhausen (1968); Schweppe (1968, 1973); Milanese and Vicino (1991) and references therein for an overview on SVOs – described in Rosa et al. (2009), which are based-upon the work in Shamma and Tu (1999).

One of the advantages of this method is that it is able to cope with linear time-varying plants. In particular, linear parameter varying (LPV) models can be used under this environment. These LPV (see Shamma (1988)) models represent nowadays a compromise between the global accuracy of nonlinear models and the straightforward controller synthesis and system analysis techniques available for LTI representations. Indeed, LPV descriptions of aircraft models have been extensively adopted to accurately model the desired dynamics over a set of predefined operating regions – see Esteban (2004); Rosa et al. (2007) and references therein.

In this paper, we provide an application example of a new FDI method based on SVOs, and address the performance of the aforementioned technique when applied to the detection of faults in an aircraft. The performance of the approach is assessed by simulation, by deliberately generating faults in the aircraft model. The key criteria of this evaluation are the time required to diagnose a failure, and the robustness of the method against model uncertainty and exogenous disturbances.

The remainder of this paper is organized as follows: Section 2 provides some background material on SVOs; Section 3 describes the robust FDI approach for LPV systems that is going to be used throughout this paper; Section 4 presents the LPV longitudinal model of an aircraft; simulation results for this plant are presented in Section 5; finally, Section 6 summarizes some conclusions.

2. BACKGROUND MATERIAL

As described in Section 3, the problem of "disqualifying" dynamic models of a system can be tackled using set-valued observers (SVOs). This type of observers was developed for linear time-varying systems and later on extended to uncertain plants (see Rosa et al. (2009)). In this section, we provide some background material on SVOs and on its applicability to fault detection and isolation, required for the remainder of the paper.

Consider that the non-faulty plant can be represented by an uncertain (possibly time-varying) discrete-time linear system, with uncertain initial conditions, and excited by bounded but unknown exogenous disturbances, *i.e.*,

$$\begin{cases} x(k+1) = A(k)x(k) + A_\Delta(k)x(k) + L_d(k)d(k) \\ \qquad\qquad +B(k)u(k) \\ \quad y(k) = C(k)x(k) + n(k), \end{cases} \tag{1}$$

where $x(0) = x_0$, $x_0 \in X(0)$, $d(k)$ with $|d(k)| = \max_i |d_i(k)| \leq 1$ are the disturbances, $n(k)$ with $|n(k)| = \max_i |n_i(k)| \leq \bar{n}$ is the sensors noise, $u(k)$ is the control input, $y(k)$ is the measured output, $x(k)$ is the state of the system and $X(0) := \mathrm{Set}(M_0, m_0)$, where

$$\mathrm{Set}(M, m) := \{q : Mq \leq m\} \tag{2}$$

represents a convex polytope. Moreover, let $x(k) \in \mathbb{R}^n$, $d(k) \in \mathbb{R}^{n_d}$, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$.

Furthermore, assume that [1]

$$A_\Delta(k) = A_1(k)\Delta_1(k) + A_2(k)\Delta_2(k) + \ldots + A_{n_A}\Delta_{n_A}(k),$$

for $|\Delta_i(k)| \leq 1, i = 1, \ldots, n_A$. The scalars $\Delta_i(k)$, $i = \{1, \ldots, n_A\}$, represent parametric uncertainties, while the matrices A_i, $i = \{1, \ldots, n_A\}$, are the directions which those uncertainties act upon.

An SVO attempts to generate the smallest $X(k+1)$ such that $x(k+1) \in X(k+1)$, based upon (1) and with the additional knowledge that $x(k) \in X(k), x(k-1) \in X(k-1), \cdots, x(k-N) \in X(k-N)$ for some finite N. Moreover, it also requires that for all $x \in X(k+1)$, the observations are compatible with (1). In this paper, we use the procedure introduced in Shamma and Tu (1999) for time-varying discrete-time linear dynamic systems, which was later on extended to uncertain plants in Rosa et al. (2009).

For plants with uncertainties, the set $X(k+1)$ is, in general, non-convex, even if $X(k)$ is convex. Thus, it cannot be represented by (2). One solution to this problem is to overbound this set by another, denoted by $\hat{X}(k+1)$, which is going to be described in the sequel.

Let $v_i, i = 1, \ldots, (Nn_A)^2$, for some positive scalar N, denote a vertex of the hyper-cube $H := \left\{\delta \in \mathbb{R}^{Nn_A} : |\delta| \leq 1\right\}$, where $v_i = v_j \Leftrightarrow i = j$. Then, we denote by $\hat{X}_{v_i}(k+1)$ the set of points $x(k+1)$ that satisfy (1) with $[\Delta(k)^{\mathrm{T}}, \cdots, \Delta(k-N+1)^{\mathrm{T}}]^{\mathrm{T}} = v_i$ and with $x(k) \in \hat{X}(k)$, $\cdots$, $x(k-N+1) \in \hat{X}(k-N+1)$. Further define

$$\hat{X}(k+1) := \mathrm{co}\left\{\hat{X}_{v_1}(k+1), \cdots, \hat{X}_{v_{(Nn_A)^2}}(k+1)\right\},$$

where $\mathrm{co}\{p_1, \ldots, p_m\}$ is the convex hull of $p_1, \ldots, p_m$.

Since, as previously mentioned, $X(k+1)$ is, in general, non-convex even if $X(k)$ is convex, we are going to use $\hat{X}(k+1)$ to overbound the set $X(k+1)$. An illustration for the case $n_A = 1$, $N = 1$, is depicted in Fig. 1.

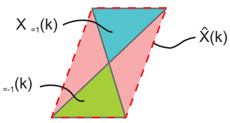

Fig. 1. Convex hull, $\hat{X}(k)$, of the sets generated by the solutions to (1) with $n_A = 1$, $N = 1$ and for $\Delta = 1$ and $\Delta = -1$.

It is important to stress that, under certain conditions, the set $\hat{X}(k+1)$ contains $X(k+1)$, as demonstrated next.

***Proposition** 1.* Consider a system described by (1) and let $\tilde{A}_m^k(\Delta) = (A(k) + A_\Delta(k)) \cdots (A(k-m) + A_\Delta(k-m))$. Further define $H(\tilde{A}_m^k)(\Delta^*)$ as the Hessian matrix of $\tilde{A}_m^k$ with respect to Δ, evaluated at Δ^*. If $H(\tilde{A}_m^k)(\Delta^*) \geq 0$ for all Δ^* such that $|\Delta^*| \leq 1$ and for all $m = 0, \cdots, N-1$, then $X(k) \subseteq \hat{X}(k)$ for all $k \in \{0, 1, 2, \cdots\}$.

Proof: Denote by $X_{v_i}(k+1)$ the set of points $x(k+1)$ that verify (1) with $A_\Delta \equiv A_{v_i}$ and with $x(k) \in X(k)$. Further define

$$X^*(k+1) := \mathrm{co}\left\{X_{v_1}(k+1), X_{v_2}(k+1), \cdots, X_{v_{n_A}}(k+1)\right\}.$$

Then, it is clear that $X^*(k) \subseteq \hat{X}(k)$ for all $k \in \{0, 1, 2, \cdots\}$. Hence, we only have to prove that $X(k) \subseteq X^*(k)$. However, this comes naturally from the fact $\tilde{A}_m^k : \mathbb{R}^{Nn_a} \to \mathbb{R}^{n^2}$ is a convex map of the variables $\Delta(k)$, $\cdots$, $\Delta(k-N+1)$. $\square$

[1] Notice that this method can be used to model several types of uncertainty, such as the uncertainty in the geometry or mass of the aircraft.

Although this approach adds some conservatism to the solution, it possesses the valuable property summarized in Proposition 2.

Proposition 2. Suppose that a system described by (1) with $x(0) = x_0$ and $u(k) = 0, \forall k$, verifies, for sufficiently large N^*,

$$\gamma_N := \max_{\substack{\Delta(k), \cdots, \Delta(k+N) \\ |\Delta(m)| \leq 1, \forall_m \\ k \geq 0}} \left\| \Pi_{j=k}^{k+N} \mathcal{A}(j) \right\| < 1,$$

for all $N \geq N^*$, and where

$$\mathcal{A}(j) := \left[A(j) + \sum_i A_i(j)\Delta_i(j) \right].$$

Then, $\hat{X}(k)$ cannot grow without bound.

Proof: Consider the smallest hyper-cubes, denoted by $\Psi(1), \Psi(2), \cdots, \Psi(m)$, that contain the sets $\hat{X}(1)$, $\hat{X}(2)$, $\cdots$, $\hat{X}(m)$, respectively. Let $N \geq N^*$. Then, an SVO can be synthesized to generate the sets $\Psi(1), \Psi(2), \cdots, \Psi(m)$, using the following inequality:

$$|x(k+N)| \leq \gamma_N |x(k)| + \delta_N, \tag{3}$$

where

$$\delta_N = \max_{d(k), \cdots, d(k+N-1)} |A^{N-1}Ld(k) + \cdots + Ld(k+N-1)|.$$

Notice that it suffices to show that the sequence $\Psi(1)$, $\Psi(2)$, $\cdots$, $\Psi(m)$ does not grow without bound, since it contains $\hat{X}(1), \hat{X}(2), \cdots, \hat{X}(m)$. Given that $\gamma_N < 1$ by assumption and that $|\delta_N| < \infty$ since $|d| < \infty$, the sets defined by (3) cannot grow without bound, which concludes the proof. □

Remark 1: Notice that, in order to guarantee that $\hat{X}$ does not grow without bound, an SVO should use the N most recent estimates. In other words, the estimation of $\hat{X}(k+N)$ should take into account the fact that $x(k) \in \hat{X}(k), x(k+1) \in \hat{X}(k+1), \cdots, x(k+N-1) \in \hat{X}(k+N-1)$. The exact description of the SVO is omitted here due to lack of space. ⋄

As a final remark, the uncertainty in $B(k)$ and $C(k)$ can also be handled by the algorithm – the interested reader is referred to Rosa et al. (2009) for further details.

3. FAULT DETECTION AND ISOLATION USING SVOS

We are now ready to state the main results that are going to be used for the fault detection and isolation of a linear parameter varying (LPV) model of an aircraft. We start by analyzing the applicability of the SVOs for fault detection (FD), and extend it to fault detection and isolation.

The following proposition is required in what follows to guarantee that there will be no false alarms.

Proposition 3. Consider a non-faulty plant described by (1) and a corresponding SVO, as in Section 2. Then, if $\hat{X}(k) = \emptyset$ for some $k \geq 0$, a fault has occurred at some time-instant k_0, where $k_0 \leq k$.

Proof: If $\hat{X}(k)$ is empty for some k, then the observations are not compatible with the model of the plant. Since we assume that the non-faulty plant can be described by (1), we conclude that a fault has occurred. □

The architecture depicted in Fig. 2, together with the result in Proposition 3, can be used to address the problem of fault detection for discrete-time linear time-varying plants. In reference to Fig. 2, the FD filter is composed of an SVO and a logic block, which decides whether or not a fault has occured, according to the emptiness or not of the set-valued estimate of the state, at each sampling time.

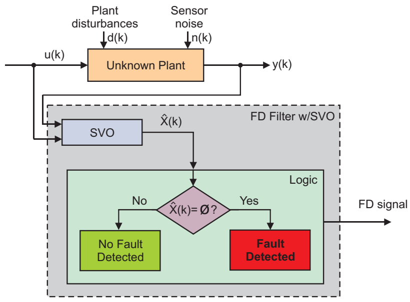

Fig. 2. Fault Detection (FD) architecture for uncertain plants using a Set-Valued Observer (SVO).

The fault isolation techniques available in the literature try to identify a very precise faulty behavior, after a general fault is detected. This means that, unlike an FD filter, an FDI filter should not only be able to detect a faulty behavior of the plant, but also to provide information regarding its whereabout.

The SVOs are also suitable for fault isolation, as long as the corresponding model of the fault has been considered during the design. Using the aforementioned results, an SVO can be designed for a particular fault that is required to be properly isolated. Two additional SVOs, besides the faults isolation SVOs, are synthesized:

(1) one SVO for the non-faulty (probably uncertain and time-varying) plant – referred to as *nominal SVO*;
(2) one SVO that is able to handle the faulty and non-faulty plant – referred to as *robust SVO*.

The *nominal SVO* is used for fault detection only. Notice that this SVO produces state estimates valid not only for the nominal model of the plant, but also for any plant belonging to the family of admissible plants. As previously described, the set-valued estimate for the state of the plant, obtained using this observer, is non-empty, if the plant does not present a faulty behavior. If the state estimate of the nominal SVO is the empty set, a fault has occurred. Hence, the fault isolation SVOs are initialized with the state estimate provided by the *robust SVO*. A fault is completely isolated whenever only one of the fault isolation SVO has a non-empty set-valued state estimation.

Remark 2: Despite the designation of the filters, all the SVOs should be robust against model uncertainty, including the nominal and the faults isolation SVOs. For example, the nominal SVO should take into account uncertainty in the geometry and mass of the aircraft, and in the readings that are acquired from the sensors. ⋄

The overall architecture of the FDI filter with SVOs is depicted in Fig. 3. It should be stressed that the FD filters with SVOs that are designed for specific faults, are only initialized with the set-valued state estimate of the robust SVO when a fault is detected by the nominal FD filter.

This approach, however, may lead to some practical problems since the set-valued state estimate provided by the aforementioned *robust SVO* can be very *large*, which, in turn, leads to a *very long* fault isolation period. A solution to overcome this issue is presented next.

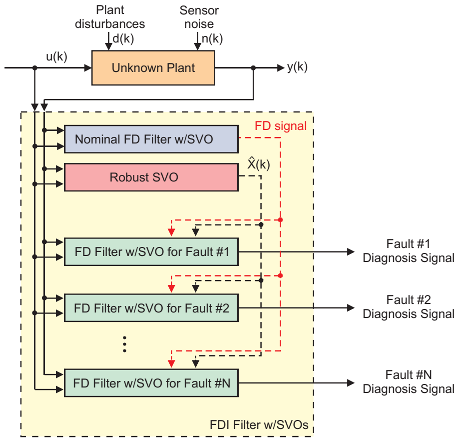

Fig. 3. Fault Detection and Isolation (FDI) architecture for uncertain plants using Set-Valued Observers (SVOs).

Consider, for instance, a loss-of-effectiveness type of fault in an actuator. This fault can be modeled by multiplying the actuator input by a constant $\lambda \in [0,\, 1]$. Therefore, consider an SVO, as described in Section 2, designed for a plant with this type of uncertainty. Then, such an SVO would validate observations from a model with any value of $\lambda \in [0,\, 1]$ and, in particular, for $\lambda = 1$, which corresponds to the nominal plant. Hence, the set-valued estimate of this fault isolation filter need not be initialized with that of the *robust SVO* whenever a fault is detected.

Therefore, in reference to Fig. 3, one can use only the fault isolation filters, if each of these filters includes the nominal model of the plant as one of the admissible models. A fault is then isolated when only one of the isolation filters provides a non-empty estimate of the state of the plant.

4. AIRCRAFT LONGITUDINAL LPV MODEL

As previously mentioned, in this paper we study the applicability and the performance of a novel FDI strategy to an aircraft linear parameter varying (LPV) Shamma (1988) longitudinal model. These LPV models are, in general, time-varying and represent nowadays a compromise between the global accuracy of nonlinear models and the straightforward controller synthesis and system analysis techniques available for linear time-invariant (LTI) descriptions.

The dynamics of an aircraft are highly nonlinear, and depend on several (time-varying) parameters, such as the dynamic pressure and the aerodynamic coefficients. However, in constant altitude steady-state flight, these dynamics are well-described by LPV models, which depend upon the airspeed. In particular, consider the aircraft LPV longitudinal model presented in Fujimori and Ljung (2006). This model can be described by the following linearized equations:

$$\begin{aligned} \frac{du}{dt} - X_u u + g\cos\Theta_o\theta &= 0, \\ -Z_u u + V\frac{d\alpha}{dt} - Z_\alpha\alpha + (V+Z_q)q + +g\sin\Theta_o\theta &= Z_{\delta_e}\delta_e, \\ -M_u u - M_{\dot\alpha}\frac{d\alpha}{dt} - M_\alpha\alpha + \frac{dq}{dt} - M_q q &= M_{\delta_e}\delta_e, \\ \frac{d\theta}{dt} &= q. \end{aligned} \tag{4}$$

The longitudinal states are the forward airspeed, u (which should not be confused with the control input in (1)), the pitch angle, θ, the angle-of-attach, α, and the pitch rate, q. The parameters of the model are the stability and control derivatives (described in the sequel), the magnitude of the gravity vector, g, the pitch trimming angle, Θ_o, and the airspeed, V, *i.e.*, the magnitude of the velocity of the aircraft relative to the fluid. Moreover, δ_e is the deviation of the elevator angle.

As explained in detail in Fujimori and Ljung (2006), by defining $x(t)$ and $y(t)$ as

$$x(t) := \begin{bmatrix} u(t) \\ \theta(t) \\ \alpha(t) \\ q(t) \end{bmatrix}, \quad y(t) := \begin{bmatrix} u(t) \\ \theta(t) \\ \alpha(t) \end{bmatrix},$$

the dynamics in (4) can be rewritten as the following continuous-time LPV model:

$$\begin{cases} \dfrac{d}{dt}x(t) = A(V,\xi(V))x(t) + B(V,\xi(V))\delta_e(t), \\ y(t) = Cx(t), \end{cases} \tag{5}$$

where

$$A(V,\xi(V)) = [a_{ij}],$$

$$\begin{aligned} &a_{11} = X_u, && a_{12} = -g\cos\Theta_o, && a_{13} = X_\alpha, \\ &a_{14} = 0, && a_{21} = 0, && a_{22} = 0, \\ &a_{23} = 0, && a_{24} = 1, && a_{31} = \frac{Z_u}{V}, \\ &a_{32} = -\frac{g\sin\Theta_o}{V}, && a_{33} = \frac{Z_\alpha}{V}, && a_{34} = 1 + \frac{Z_q}{V}, \\ &a_{41} = M_u + M_{\dot\alpha}\frac{Z_u}{V}, && a_{42} = -M_{\dot\alpha}\frac{g\sin\Theta_o}{V}, \\ &a_{43} = M_\alpha + M_{\dot\alpha}\frac{Z_\alpha}{V}, && a_{44} = M_q + M_{\dot\alpha}\left(1 + \frac{Z_q}{V}\right), \end{aligned}$$

$$B(V,\xi(V)) = \begin{bmatrix} 0 \\ 0 \\ \frac{Z_{\delta_e}}{V} \\ M_{\delta_e} + M_{\dot\alpha}\frac{Z_{\delta_e}}{V} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The stability and control derivatives (SCDs) are concatenated in vector $\xi(V)$, *i.e.*,

$$\xi(V) = [X_u,\, X_\alpha,\, Z_u,\, Z_\alpha,\, Z_q,\, M_u,\, M_\alpha,\, M_{\dot\alpha},\, M_q,\, Z_{\delta_e},\, M_{\delta_e}].$$

For a nominal airspeed of $V_o = 150$ m/s, the numeric values of the SCDs are summarized in Table 1.

Table 1. Stability and control derivatives for $V_o = 150$ m/s.

SCD	Value	SCD	Value
X_u	-0.0298	X_α	12.609
Z_u	-0.3065	Z_α	-161.54
Z_q	-1.5464	M_u	0.0013
M_α	-7.9295	M_q	-1.8485
$M_{\dot\alpha}$	0.1167	Z_{δ_e}	-11.374
M_{δ_e}	-5.9544		

The LPV model in (5) is going to be used in simulation to represent the dynamics of the aircraft and to design the SVOs for the FDI method described in Section 3.

5. SIMULATIONS

This section presents a series of simulations that illustrate the applicability of the SVOs in fault detection and isolation.

We use the aircraft model for the longitudinal axis, presented in Section 4, discretized with a sampling period $T_s = 200$ ms. Using this model and the results summarized in Section 2, an SVO was designed for this plant.

Remark 3: Applying Proposition 2, it can be seen that by using the 12 most recent state estimates, the SVOs are guaranteed to converge. ◇

Thereafter, a fault detection filter as in Section 3 was synthesized in order to diagnose faults in the longitudinal dynamics of the aircraft. For the simulations, both the exogenous disturbances, $d(k)$, and the sensors noise, $n(k)$, were generated using a random uniform distribution. Moreover, four fault isolation filters were also designed using the approach in Section 3, in order to isolate the following failures in the aircraft:

(1) FDI #1: loss-of-effectiveness (LOE) in the forward velocity, u, sensor;
(2) FDI #2: LOE in the pitch angle, θ, sensor;
(3) FDI #3: LOE in the angle-of-attach, α, sensor;
(4) FDI #4: LOE in the elevator, δ_e.

The FDI architecture used is depicted in Fig. 4. In this case, the FDI filters were designed so that loss-of-effectiveness type of faults can be diagnosed. Thus, as explained in detail in Section 3, the FDI filters need not be reset whenever a fault is detected by the nominal filter.

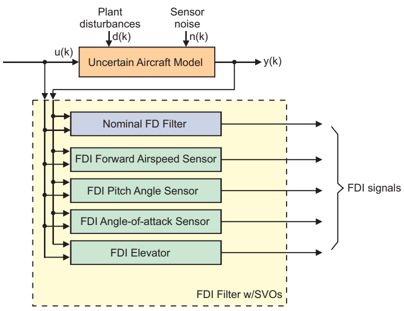

Fig. 4. FDI architecture using SVOs for the aircraft longitudinal model.

An LQG controller was designed for the aircraft linearized model, about the nominal airspeed of $V_o = 150$ m/s, in order to be able to generate faults in the actuator.

Two different scenarios are going to be analyzed in the sequel. The first one consists in generating abrupt (or hard) faults in the sensors/actuator. We start by considering a hard fault in the elevator. In particular, for $t \geq 20$ s, the elevator becomes stuck at zero, *i.e.*, $\delta_e(t) = 0$ for $t \geq 20$ s. For this configuration, the average results for 5 Monte-Carlo runs are depicted in Fig. 5. The faults were detected in less than 3 iterations, *i.e.*, in less than 600 ms, and isolated in less than 1 s.

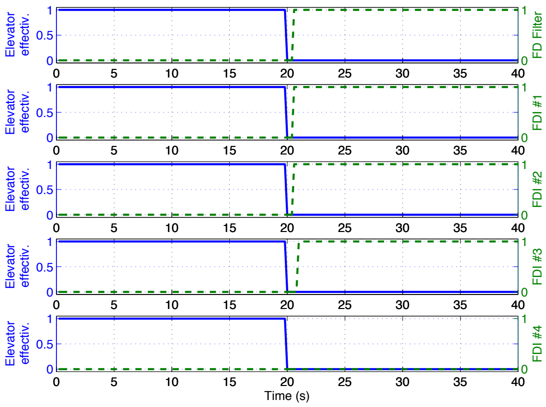

Fig. 5. Hard fault in the elevator of the aircraft. The results shown were obtained by averaging 5 Monte-Carlo runs. After nearly 1 s, the only FDI filter that is able to explain the observations is FDI #4. Therefore, the fault in the elevator is isolated in nearly 1 s.

A similar trial was tested for a hard failure in the forward speed sensor of the aircraft. Suppose that, for $t \geq 20$ s, the effectiveness of the forward speed sensor is decreased by 40%, *i.e.*, the reading acquired from the sensor corresponds to 60% of the *true* forward speed of the aircraft. For this case, the results obtained by averaging 5 Monte-Carlos runs are illustrated in Fig. 6. The faults were detected and isolated in less than 2 iterations, which is equivalent to 400 ms. In fact, in most cases, only one iteration was required to isolate this fault.

Remark 4: In order to give some insight regarding why the failures in the forward speed sensor are, in these simulations, more quickly detected and isolated than the faults in the elevator, we stress that the changes in the forward speed affect not only the state trajectory $u(\cdot)$, but also the dynamics of the model. To see this, notice that $A(\cdot,\cdot)$ and $B(\cdot,\cdot)$ in (5) depend upon the airspeed, $V(\cdot)$, which is obviously related to $u(\cdot)$. $\diamond$

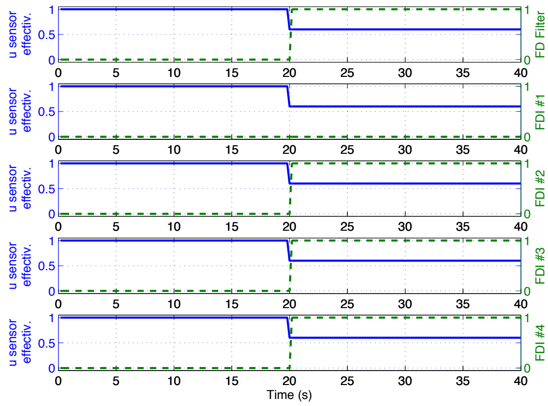

Fig. 6. Hard fault in the forward speed sensor. The fault in the forward speed sensor is isolated in nearly 400 ms.

It should be noticed, however, that hard faults are, in general, "easier" for detection. Indeed, for the second scenario, we consider smooth (or soft) faults in the sensors/actuator, thus representing more realistic failures.

Suppose that the effectiveness of the elevator suffers the variation depicted in Fig. 7, *i.e.*, the effectiveness of the actuator decreases linearly during 2 s. In this case, the FD filter with SVOs takes nearly 600 ms to detect the fault, as shown in Fig. 7. Moreover, the fault is isolated in 2 s. In comparison with the previous scenario (hard failures), the FDI system requires more time to detect and isolate the faults, as expected.

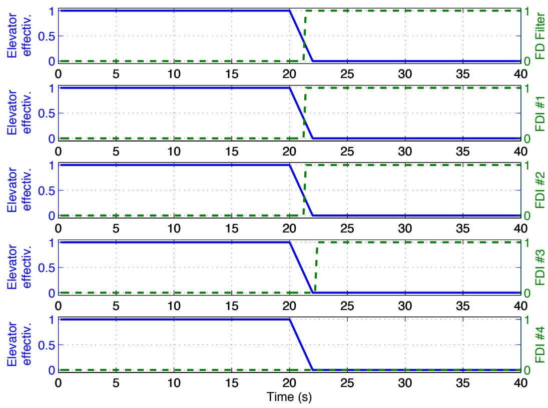

Fig. 7. Soft fault in the elevator of the aircraft. The fault in the elevator is isolated in nearly 2 s.

The results for a soft fault in the forward speed sensor are depicted in Fig. 8. In this situation, the results are not significantly affect by the smoothness of the fault. Indeed, only 2 iterations are required to isolate the fault. The same reasoning as in Remark 4 applies to this case.

Several Monte-Carlo simulations with faults occurring in the pitch angle and angle-of-attack sensors were also

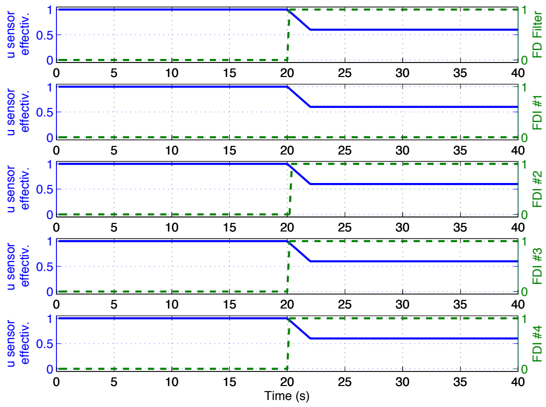

Fig. 8. Soft fault in the forward speed sensor. The fault in the forward speed sensor is isolated in nearly 400 ms.

conducted, leading to similar results, though they are omitted here due to lack of space.

6. CONCLUSIONS

This paper illustrated the applicability of a novel methodology for fault detection and isolation (FDI) based on set-valued observers (SVOs), with the example of a linear parameter varying (LPV) aircraft longitudinal model. The adopted approach relies on the set-valued state estimates of the SVOs to validate or falsify the set of observations. These SVOs are designed in such a way that model uncertainty and disturbances can be accounted for. Hence, *false alarms* due to such perturbations are avoided.

The simulation results show that the detection and isolation of the faults take, in general, only a few iterations. It was also noticed that, as expected, abrupt faults are easier to detect than smooth faults, using this approach. The results obtained show that the methodology derived holds considerable promise for practical applications.

REFERENCES

Besançon, G. (2003). High-gain observation with disturbance attenuation and application to robust fault detection. *Automatica*, 39, 1095–1102.

Blanke, M., Izadi-Zamanabadi, R., Bogh, S., and Lunau, C. (1997). Fault tolerant control systems – a holistic view. *Control Engineering Practice*, 5(5), 693–702.

Blanke, M., Staroswiecki, M., and Wu, N. (2001). Concepts and methods in fault-tolerant control. In *Proceedings of the American Control Conference, Arlington, VA, USA*.

Bokor, J. and Balas, G. (2004). Detection filter design for lpv systems – a geometric approach. *Automatica*, 40, 511–518.

Collins, E.G.J. and Tinglun, S. (2001). Robust l_1 estimation using the popov–tsypkin multiplier with application to robust fault detection. *Int. J. Control*, 74(3), 303–313.

Edelmayer, A., Bokor, J., and Keviczky, L. (1994). An h_∞ approach to robust detection of failures in dynamical systems. In *Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista, FL, USA*, volume 3, 3037–3039.

Esteban, A.M. (2004). *Aircraft Applications of Fault Detection and Isolation Techniques*. Ph.D. thesis, University of Minnesota.

Frank, P. and Ding, X. (1994). Frequency domain approach to optimally robust residual generation. *Automatica*, 30(5), 789–804.

Frank, P. and Ding, X. (1997). Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *Journal of Process Control*, 7(6), 403–424.

Fujimori, A. and Ljung, L. (2006). Model identification of linear parameter varying aircraft systems. *J. of Aerospace Engineering*, 28, 337–346.

Isermann, R. (1997). Supervision, fault-detection and fault-diagnosis methods - an introduction. *Control Engineering Practice*, 5(5), 639–652.

Longhi, S. and Moteriù, A. (2009). Fault detection for linear periodic systems using a geometric approach. *IEEE Trans. on Automatic Control*, 54(7), 1637–1643.

Mangoubi, R., Appleby, B., Verghese, G., and Vander Velde, W. (1995). A robust failure detection and isolation algorithm. In *Proceedings of the 34th Conference on Decision and Control, New Orleans, USA*, volume 3, 2377–2382.

Marcos, A., Ganguli, S., and Balas, G. (2005). An application of $\mathcal{H}_\infty$ fault detection and isolation to a transport aircraft. *Control Engineering Practice*, 13, 105–119.

Massoumnia, M.A. (1986). Geometric approach to the synthesis of failure detection filters. *IEEE Trans. on Automatic Control*, 31(9), 839–846.

Mattone, R. and De Luca, A. (2006). Relaxed fault detection and isolation: An application to a nonlinear case study. *Automatica*, 42, 109–116.

Meskin, N. and Khorasani, K. (2009). Fault detection and isolation of discrete-time markovian jump linear systems with application to a network of multi-agent systems having imperfect communication channels. *Automatica*, 45, 2032–2040.

Milanese, M. and Vicino, A. (1991). Optimal estimation theory for dynamic systems with set membership uncertainty: An overview. *Automatica*, 27(6), 997–1009.

Narasimhan, S., Vachhani, P., and Rengaswamy, R. (2008). New nonlinear residual feedback observer for fault diagnosis in nonlinear systems. *Automatica*, 44, 2222–2229.

Niemann, H. and Stoustrup, J. (2001). Fault diagnosis for non-minimum phase systems using h_∞ optimization. In *Proceedings of the American Control Conference, Arlington, VA, USA*.

Patton, R. and Chen, J. (1997). Observer-based fault detection and isolation: Robustness and applications. *Control Engineering Practice*, 5(5), 671–682.

Rosa, P., Silvestre, C., Cabecinhas, D., and Cunha, R. (2007). Autolanding controller for a fixed wing unmanned air vehicle. In *AIAA Guidance and Control Conference*.

Rosa, P., Silvestre, C., Shamma, J., and Athans, M. (2009). Multiple-model adaptive control with set-valued observers. In *Proceedings of the 48th IEEE Conference on Decision and Control*.

Schweppe, F. (1968). Recursive state estimation: Unknown but bounded errors and system inputs. *IEEE Trans. on Automatic Control*, 13(1), 22–28.

Schweppe, F. (1973). *Uncertain Dynamic Systems*. Prentice-Hall, USA.

Shamma, J. and Tu, K.Y. (1999). Set-valued observers and optimal disturbance rejection. *IEEE Transactions on Automatic Control*, 44(2), 253–264.

Shamma, J. (1988). *Analysis and Design of Gain Scheduled Control Systems*. Ph.D. thesis, MIT, Mass., USA.

Wang, D., Wang, W., and Shi, P. (2009). Robust fault detection for switched linear systems with state delays. *IEEE Trans. on Automatic Control*, 39(3), 800–805.

Willsky, A. (1976). A survey of design methods for failure detection in dynamic systems. *Automatica*, 12, 601–611.

Witsenhausen, H. (1968). Sets of possible states of linear systems given perturbed observations. *IEEE Trans. on Automatic Control*, 13(5), 556–558.